Report of task T2.1
Specifications of a data model for a graphic editor development supporting physical structural analysis

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Introduction

The different functionalities, needed for the physical structural analysis of a mechatronic system in view of its design, require a formalism that enables the implementation of a physical model (so based on a physical description of the system). Furthermore this physical model necessitates a sufficiently detailed granularity of description showing the elementary energy phenomena, the couplings and the structure of the energy exchanges between the phenomena in the system. Also, these functionalities require to keep this description level all along the simulation process from the input of the model to the structural and numerical results and analyses.

For instance the mass/spring/damper model given in Fig. 1 is represented at the elementary level of description by the following equations:

$$\begin{cases} p = \mathrm{m}v & \text{for the mass in motion -kinetic phenomena-} \\ F_{\mathrm{r}} = \mathrm{k}x & \text{for the spring in traction/compression -potential phenomena-} \\ F_{\mathrm{a}} = \mathrm{b}v & \text{for the damper -dissipative phenomena-} \\ \dot{p} + F_{\mathrm{r}} + F_{\mathrm{a}} = 0 & \text{Newton's second law} \end{cases}$$

that gather the behavior laws and the power conserving law of the model. This is the required information level for the physical structural analysis. On the contrary a more "global" description, and generally used as the model input for simulation, is given by the equation:

$$\mathrm{m}\ddot{x} + \mathrm{b}\dot{x} + \mathrm{k}x = 0$$

This equation has not the minimum required level of information for the physical structural analysis.

The issue for the physical structural analysis, the model inversion, and so, the design of mechatronic systems, is to be able, in a representation formalism, to keep the information details required in the description of the model.

From recent works it has been established that this formalism: (i) is not a compiler but the MMI (Man-Machine Interface) issue, cannot be completely supported by Modelica and has, preferably, to be graphical.

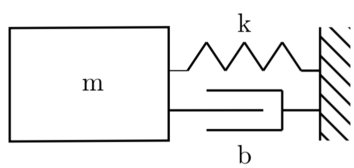

Figure 1: Mechanical example

This document is organized in five parts. The first part (Sec. 1) introduces the definitions of the basic concepts for modelling physical systems and used in the physical structural analysis. The second part (Sec. 2) presents the definitions of the physical structural analysis concepts. The third part (Sec. 3) gives the criteria expressed in the methodology for the structural invertibility verification and the mathematical differentiability conditions for the output specifications. The fourth part (Sec. 4) presents the constraints for the equation formulation corresponding to inverse model of minimal order. Finally the fifth and last part (Sec. 5) describes the data model corresponding to the previously defined concepts in view of developing the graphic formalism and editor.

Since bond graph language is well adapted to the different concepts presented in this document and that there already exists a bond graph library (BondLib developed at ETH) whose the elements are described in Modelica language, the starting point of the developments will be based on this language and this library.

Contents

1 Basic concepts for modelling physical systems

1.1 Definitions

Model of knowledge [Bor92]: A *model of knowledge* is a set of equations established on the basis of general physical models corresponding to the fundamental laws of physics, chemistry, biology,... and for which the variables have a straightforward physical interpretation : temperature, pressure, current, acceleration, force,... The models of knowledge have more significant insight than the *models of representation* which do not generally enable a physical interpretation of the studied phenomena, or, than the *input/output models* which correspond to "black box-type" models. A model of knowledge contains all the physical information necessary for a physical structural analysis on the studied system. To a large extent the *bond graph representation* is a graphical representation of a model of knowledge since it contains all the details about the physical information.

Finally a model of knowledge is obtained from the behavior laws of the physical phenomena modeled and from the conservation laws (Newton's second law, Kirchhoff laws, first principle,...)

Behavior laws: A *behavior law* is a mathematical relation characterizing a physical phenomenon and generally established experimentally (relation between momentum and velocity for a mass in translation, between charge and voltage at a capacity ports, Ohm law for an electrical resistance,...)

Conservation law of physics: A *conservation law* of physics is a net balance between energies, powers or power variables, usually established in a conjectural manner and for which no counter-example has been found in its validity domain.

Power variables, energy variables: The *power variables* and the *energy variables* are key variables in the modeling of physical dynamic systems. They are analogous between the different physical domains (Tab. 1). The power variables are of two types (*effort* e and *flow* f variables) whose the product defines the power $\mathcal{P} = e \cdot f$. The energy variables are state variables for the different elementary energy storage phenomena in the state function sens. These variables are extensive variables that represent the integrals of power variables (Tab. 1).

Table 1: Analogy of power and energy variables between the different physical domains

	Effort e	**Flow f**	***Generalized momentum*** $p = \int_0^t e(\tau)\mathrm{d}\tau + \mathrm{p_0}$	***Generalized displacement*** $q = \int_0^t f(\tau)\mathrm{d}\tau + \mathrm{q_0}$
Translational mechanics	Force (N)	Velocity (m/s)	linear Momentum (kg·m/s)	Displacement (m)
Rotational mechanics	Torque (Nm)	Angular velocity (rad/s)	Angular momentum (kg·m2·rad/s)	Angle (rad)
Electricity	Voltage (V)	Current (A)	Total magnet. flux (Wb)	Charge (C)
Hydraulics	Pressure (Pa)	Flow rate (m3/s)	Pressure momentum (Pa·s)	Volume (m3)
Magnetism	Magnetomotive force (A)	Derivative of magnet. flux (Wb/s)		Magnetic flux (Wb)
Chemistry	Chemical potential (J/mole)	Molar flow rate (mole/s)		Mole (mole)
Thermodynamics	Temperature (K)	Entropy flow rate (W/K)		Entropy (J/K)

Table 2: Examples of components that exhibit elementary physical phenomena

	Energy sources/sinks	**Energy storage**	**Energy dissipation**
Translational mechanics	Cylinder (ideal actuator)	Moving mass - Mass in a gravity field - Spring	Friction - Damper
Rotational mechanics	Drive (ideal actuator)	Rotating inertia - Torsion rod	Joint friction
Electromagnetic circuits	Voltage source - Current source	Capacitor - Windings	Resistance - Diode
Hydraulic circuits	Pump - Compressor	Hydraulic pipe - Accumulator - Cylinder chamber	Orifice - Restriction
Thermodynamic systems	Heat pump - Heat source	Thermal capacity	Conduction

Elementary physical phenomena: An *elementary physical phenomenon* corresponds, in an exclusive way, to an energy *source/sink*, to an *energy storage* or to an *energy dissipation* (see examples in Tab. 2). These phenomena will be characterized by behavior laws involving a power variable (e or f) and depending on parameters or the functions of other variables; between an energy variable and the conjugated power variable (*coenergy variable* $-e = e(q)$ or $f = f(q)-$) of a same physical domain for the storage energy; between two power variables ($g(e, f) = 0$) of a same physical domain for dissipative phenomena. Concerning energy storage it could be a commodity in modeling to make use of another state variable than that of energy. This state variable will be taken into account in the set of the *supplementary variables* of the model.

Multiport elementary physical phenomena: A *multiport elementary physical phenomenon* is an elementary physical phenomenon characterized by more than one behavior law involving power variables and energy variables from different physical domains. For a multiport phenomenon the number of energy variables corresponds to the number of *ports*. Finally an elementary physical phenomenon can be viewed as a *1-port elementary physical phenomenon*.

Energy transduction: An *Energy transduction* is associated with a physical coupling between two parts of a system. It can be *power conserving* if the transduction is instantaneous (without delay) and without energy loss. Otherwise it involves multiport elementary physical phenomena corresponding to dissipation or energy storage.

Modulation : A *modulation* may occur in the behavior laws associated with the elementary physical phenomena of energy source, energy dissipation or associated with a power conserving energy transduction. This modulation uses another variable than the power variables involved in the *modulated* behavior laws. Two types of modulation exist: *internal modulation* and *input modulation*. For an internal modulation the modulating variable results from a relation involving one or more power and/or energy variables of the model. Most of the time internal modulation affects a power conserving energy transduction as for instance in the transformation matrix in the multibody mechanical domain. For an input modulation the modulating variable so-called *modulating input* results from input controls of the model. A modulating input generally appears in a behavioral law associated with a dissipation phenomenon, with an energy source, or in a law of power conserving energy transduction.

Inputs and outputs: The *inputs* of a model are the variables supposed given for the model and defined as functions of time. It can be distinguished the *control inputs*, associated with the functional objectives of the corresponding system, and the *perturbation inputs*, associated with uncontrolled solicitations from the environment of the system. The *outputs* of a model correspond to variables that can be calculated from the model (state) variables. They are generally viewed as, in the most general sense, observed quantities in the system.

Calculus relations: The *calculus relations* are supplementary relations (in addition to the behavior and conservation laws) in the model. Associated with the internal modulation variables the calculus relations relate these variables to power and/or energy variables (modulation laws). Associated with the input modulation variables the calculus relations relate these variables to input control variables, output variables, and/or perturbation input variables. Finally some calculus relations involve a set of intermediary variables of calculus.

The characteristics of these calculus relations is that they do not involve power. Concerning the calculus relations between intermediary variables, they often correspond to the modeling commodity of decomposing some calculus.

These definitions emphasize a number of associations. In perspective of structural analysis it is absolutely necessary to keep a trace of both these basic concepts and their associations in the modeling formalism. These associations are presented in Fig. 2:

- the *energy structure* 1:
  - relates powers 2 to power variables 8 by the means of the conservation laws 3 and the laws of power conserving energy transduction 4,
  - involves supplementary variables 9 in the modulations associated with some power conserving energy transduction and some parameters 10 through the laws of power conserving energy transduction 3.
- the elementary physical phenomena 5:
  - involve powers 2 corresponding to a certain type 6,
  - relate power variables to energy variables 8 by the means of behavior laws 7,
  - involve supplementary variables 9 in the modulations associated with certain phenomena of energy source, of energy dissipation 7 and certain state variables other than the energy variables,
  - involve some parameters 10 in behavior laws 7,
  - involve some initial conditions 10 associat with energy storage,
  - use some intermediary calculus relations 11 for the definition of their behavior laws.
- the energy structure 1 defines the architecture of the energy exchanges between the different elementary physical phenomena 5 by the means of the conservation laws 3 and the laws of power conserving energy transduction 4.
- each power (resp. energy) 2 is associated with a pair of power variables (resp. to one or several energy variables) 8.
- some calculus relations 11:
  - between powers and energies 2, some power and energy variables 8, some supplementary variables 2 and some parameters 10 are used to express the relations between the inputs, the modulations and the outputs of the model as well as the intermediary calculus relations in order to express the laws of the elementary physical phenomena,
  - use initial conditions 10 for some supplementary integral relations.

1.2 1st example: mechanical system

The introduction example (Fig. 1) is used here to illustrate the previous basic concepts and the associations between them. In this example (Fig. 3):

- the elementary physical phenomena are: the kinetic phenomenon of the moving mass, the potential phenomenon of the deformed spring, the dissipative phenomenon in the damper,

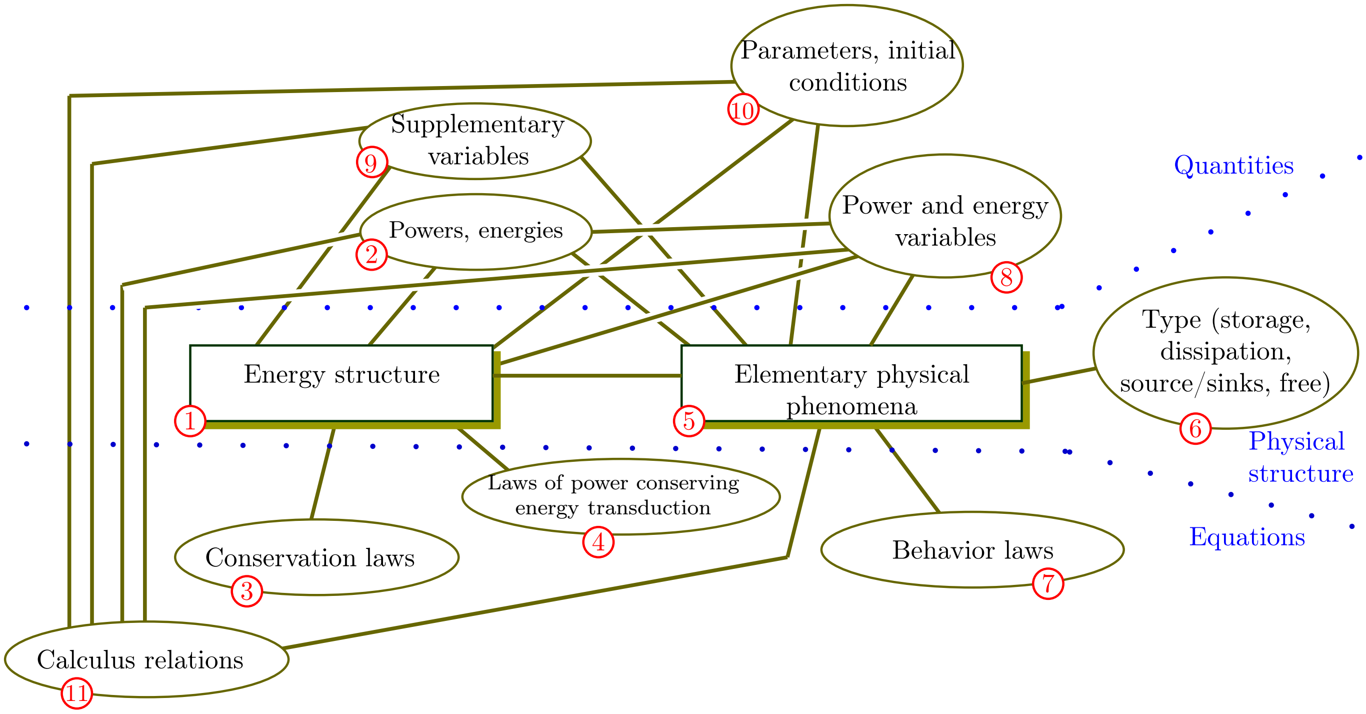

Figure 2: Association diagram between the knowledge model concepts.

- each of these phenomena is associated with a type, respectively, a storage type for the first two and a dissipation type for the third,
- the respective behavior laws are: $p = \mathrm{m}v$ for the first phenomenon, $F_\mathrm{r} = \mathrm{k}x$ for the second and, $F_\mathrm{a} = \mathrm{b}v$ for the third,
- the power variables are: the time rate of change of the linear momentum ($\dot{p}$, effort variable) and the mass velocity (v, flow variable) for the first phenomenon, the force acting on the spring (F_r, effort variable) and its elongation change rate ($\dot{x}$, flow variable) for the second phenomenon, the force acting on the damper (F_a, effort variable) and its elongation velocity (v, flow variable) for the third phenomenon,
- the energy variables are: p (generalized momentum) for the first phenomenon and x (generalized displacement) for the second phenomenon with p_0 and x_0 the respective initial conditions,
- the powers are: the power of kinetic storage energy $\mathcal{P}_\mathrm{c} = \dot{p} \cdot v$ for the first phenomenon, the power of potential energy storage $\mathcal{P}_\mathrm{p} = F_\mathrm{r} \cdot \dot{x}$ for the second phenomenon and the power of dissipation $\mathcal{P}_\mathrm{d} = F_\mathrm{a} \cdot v$ for the third phenomenon,
- the energies are: the kinetic energy $E_\mathrm{c} = \frac{1}{2}\frac{p^2}{\mathrm{m}}$ such that $\mathcal{P}_\mathrm{c} = \frac{\mathrm{d}E_\mathrm{c}}{\mathrm{d}t}$ for the first phenomenon and the potential energy $E_\mathrm{p} = \frac{1}{2}\mathrm{k}x^2$ such that $\mathcal{P}_\mathrm{d} = \frac{\mathrm{d}E_\mathrm{d}}{\mathrm{d}t}$ for the second phenomenon,
- the energy structure couples the three phenomena which exchange energy at rates represented by the previous powers,
- the conservation law associated with the energy structure is the second Newton's law that expresses a balance between the previous powers $\mathcal{P}_\mathrm{c} + \mathcal{P}_\mathrm{p} + \mathcal{P}_\mathrm{d} = 0$ at a common velocity (flow variable) $(v = \dot{x} = v)$ thus involving a force balance (effort variables) $\dot{p} + F_\mathrm{r} + F_\mathrm{a} = 0$,
- the parameters are the mass m, the stiffness k, and the damping viscous parameter,

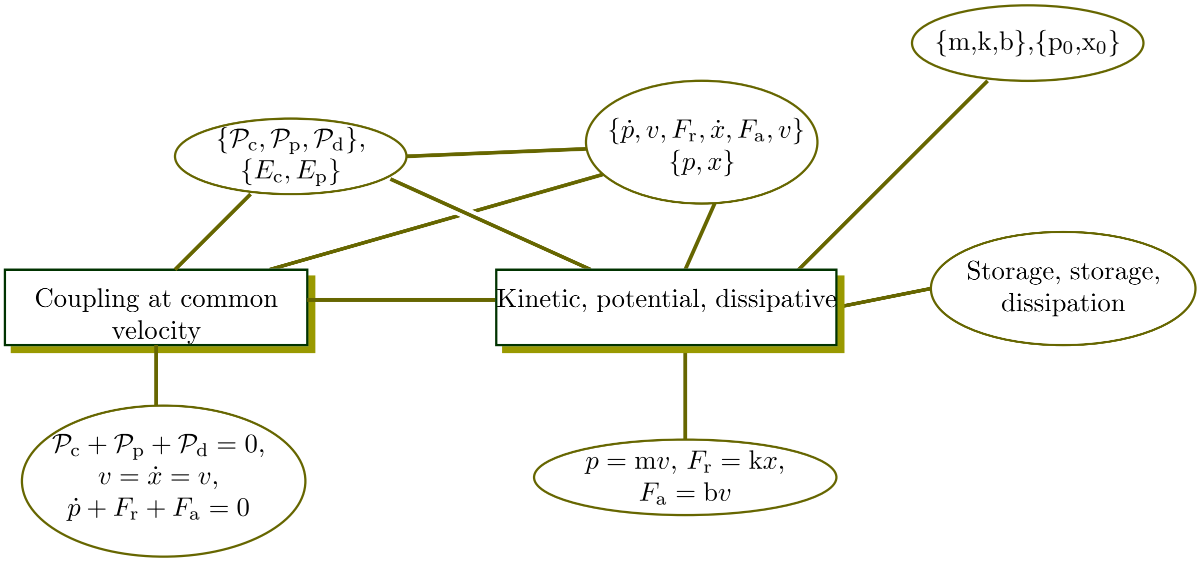

Figure 3: Concept associations for the mechanical example

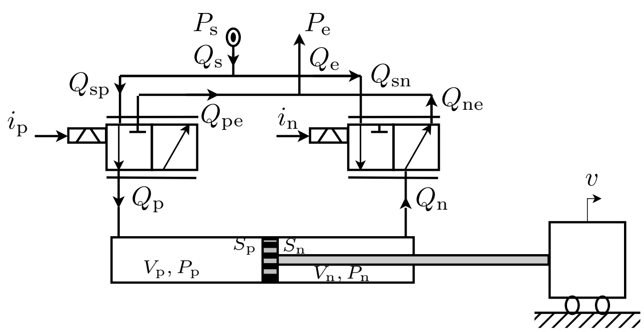

Figure 4: Scheme of an electrohydraulic system

- in this example there is no energy transduction between different physical domains, neither supplementary variable nor calculus relation.

1.3 2$^{\text{nd}}$ example: electrohydraulic system

Model

The considered electrohydraulic system is presented in Fig. 4 and is composed of:

- two proportional hydraulic servo valve 3/2 (3 orifices/2 positions) both controlled in current,
- one hydraulic double-acting jack (rod displacement controlled in both directions),
- an inertial load of mass M including the rod mass.

The volumes of both chambers are supposed constant but their time rates of variation are not neglected with respect to the volume flow rate entering the chambers.

Basic physical concepts

The different elementary physical phenomena modeled are:

- a kinetic energy storage associated with the moving mass:

- behavior law: $p = \mathrm{M}v$ (with $p = \int_0^t \dot{p}\mathrm{d}\tau + \mathrm{p}_0$ the linear momentum, M the mass, v the velocity),
- power: $\mathcal{P}_\mathrm{c} = \dot{p} \cdot v$,
- energy: E_c such that $\mathcal{P}_\mathrm{c} = \frac{\mathrm{d}E_\mathrm{c}}{\mathrm{d}t}$,
- power variables: $\dot{p}$ (effort variable) and v (flow variable),
- energy variable: p (generalized momentum) with p_0 the initial condition for p,

- an energy storage associated with the compressibility phenomena of the fluid in chamber p:
  - behavior law: $P_\mathrm{p} = \frac{B}{\mathrm{V}_{\mathrm{p}0}} q_\mathrm{p}$ (with P_p the pressure in chamber p, B the compressibility module, $V_{\mathrm{p}0}$ the volume –supposed constant– of chamber p, $q_\mathrm{p} = \int_0^t \dot{q}_\mathrm{p}\mathrm{d}\tau + \mathrm{q}_{\mathrm{p}0}$),
  - power: $\mathcal{P}_\mathrm{p} = P_\mathrm{p} \cdot \dot{q}_\mathrm{p}$,
  - energy: E_p such that $\mathcal{P}_\mathrm{p} = \frac{\mathrm{d}E_\mathrm{p}}{\mathrm{d}t}$,
  - power variables: P_p (effort variable), $\dot{q}_\mathrm{p}$ (flow variable),
  - energy variable: q_p (generalized displacement) with $\mathrm{q}_{\mathrm{p}0}$ the initial condition for q_p,

- the same for chamber n:
  - behavior law: $P_\mathrm{n} = \frac{B}{\mathrm{V}_{\mathrm{n}0}} q_\mathrm{n}$ (with P_n the pressure in chamber n, $V_{\mathrm{n}0}$ the volume –supposed constant– of chamber n, $q_\mathrm{n} = \int_0^t \dot{q}_\mathrm{n}\mathrm{d}\tau + \mathrm{q}_{\mathrm{n}0}$),
  - power: $\mathcal{P}_\mathrm{n} = P_\mathrm{n} \cdot \dot{q}_\mathrm{n}$,
  - energy: E_n such that $\mathcal{P}_\mathrm{n} = \frac{\mathrm{d}E_\mathrm{n}}{\mathrm{d}t}$,
  - power variables: P_n (effort variable), $\dot{q}_\mathrm{n}$ (flow variable),
  - energy variable: q_n (generalized displacement) with $\mathrm{q}_{\mathrm{n}0}$ the initial condition for q_n,

- an energy dissipation associated with the energy loss phenomena of the fluid in the the orifice (denoted 'sp') between the pressure source and the chamber p:
  - behavior law: $Q_\mathrm{sp} = A_\mathrm{sp}(i_\mathrm{p}^*) C_Q \sqrt{\frac{2}{\rho}\,|\Delta P_\mathrm{sp}|}\mathrm{sign}(\Delta P_\mathrm{sp})$ (with Q_sp the volume flow rate in the orifice, A_sp the area of the orifice section modulated by i_p^* a function of the current of the valve slide position, C_Q the flow coefficient, ρ the fluid specific mass, ΔP_sp the pressure drop),
  - power: $\mathcal{P}_\mathrm{sp} = \Delta P_\mathrm{sp} \cdot Q_\mathrm{sp}$,
  - power variables: ΔP_sp (effort variable), Q_sp (flow variable),

- the same for the orifice (denoted 'pe') between the chamber p and the exhaust:
  - behavior law: $Q_\mathrm{pe} = A_\mathrm{pe}(i_\mathrm{p}^*) C_Q \sqrt{\frac{2}{\rho}\,|\Delta P_\mathrm{pe}|}\mathrm{sign}(\Delta P_\mathrm{pe})$ (with Q_pe the volume flow rate in the orifice, A_pe the area of the orifice section modulated by i_p^*, ΔP_pe the pressure drop),
  - power: $\mathcal{P}_\mathrm{pe} = \Delta P_\mathrm{pe} \cdot Q_\mathrm{pe}$,
  - power variables: ΔP_pe (effort variable), Q_pe (flow variable),

- the same for the orifice (denoted 'sn') between the source and the chamber n:
  - behavior law: $Q_\mathrm{sn} = A_\mathrm{sn}(i_\mathrm{n}^*) C_Q \sqrt{\frac{2}{\rho}\,|\Delta P_\mathrm{sn}|}\mathrm{sign}(\Delta P_\mathrm{sn})$ (with Q_sn the volume flow rate in the orifice, A_sn the area of the orifice section modulated by i_n^* of function of the current of the valve slide position, ΔP_sn the pressure drop),
  - power: $\mathcal{P}_\mathrm{sn} = \Delta P_\mathrm{sn} \cdot Q_\mathrm{sn}$,
  - power variables: ΔP_sn (effort variable), Q_sn (flow variable),

- the same for the orifice (denoted 'ne') between the chamber n and the exhaust:
  - behavior law: $Q_\mathrm{ne} = A_\mathrm{ne}(i_\mathrm{n}^*) C_Q \sqrt{\frac{2}{\rho}\,|\Delta P_\mathrm{ne}|}\mathrm{sign}(\Delta P_\mathrm{ne})$ (with Q_ne the volume flow rate in the orifice, A_ne the area of the orifice section modulated by i_n^*, ΔP_ne the pressure drop),

- power: $\mathcal{P}_{\rm ne} = \Delta P_{\rm ne} \cdot Q_{\rm ne}$,
- power variables: $\Delta P_{\rm ne}$ (effort variable), $Q_{\rm ne}$ (flow variable),

- an energy source associated with the pressure source:
  - behavior law: $P_{\rm s} = \rm P$ (with P the given pressure for the source),
  - power: $\mathcal{P}_{\rm s} = P_{\rm s} \cdot Q_{\rm s}$ (with $Q_{\rm s}$ the outgoing volume flow rate of the pressure source),
  - power variables: $P_{\rm s}$ (effort variable), $Q_{\rm s}$ (flow variable),
- an energy sink associated with the tank:
  - behavior law: $P_{\rm e} = \rm P_{atm}$ (with $P_{\rm e}$ the exhaust pressure, $\rm P_{atm}$ the atmospheric pressure),
  - power: $\mathcal{P}_{\rm e} = P_{\rm e} \cdot Q_{\rm e}$ (with $Q_{\rm e}$ the ingoing volume flow rate to the tank),
  - power variables: $P_{\rm e}$ (effort variable), $Q_{\rm e}$ (flow variable).

The energy structure shows:

- a coupling between the powers $\mathcal{P}_{\rm p/piston}$ and $\mathcal{P}_{F_{\rm p/piston}}$ (hydraulic and mechanical powers developed by the pressure actions of the chamber p fluid on the piston) through a power conserving energy transduction characterized by the piston area $S_{\rm p}$ ($S_{\rm p}P_{\rm p} = F_{\rm p/piston}$ and $\frac{{\rm d}V_{\rm p}}{{\rm d}t} = S_{\rm p}v$ with $\frac{{\rm d}V_{\rm p}}{{\rm d}t}$ the time rate of change of the chamber p volume (not neglected): $\mathcal{P}_{\rm p/piston} = \mathcal{P}_{F_{\rm p}/piston}$,
- a coupling between the powers $\mathcal{P}_{\rm n/piston}$ and $\mathcal{P}_{F_{\rm n/piston}}$ (hydraulic and mechanical powers developed by the pressure actions of the chamber n fluid on the piston) through a power conserving energy transduction characterized by the difference of both the piston and the rod section areas $S_{\rm n}$ ($S_{\rm n}P_{\rm n} = F_{\rm n/piston}$ and $\frac{{\rm d}V_{\rm n}}{{\rm d}t} = S_{\rm n}v$ with $\frac{{\rm d}V_{\rm n}}{{\rm d}t}$ the time rate of change of the chamber n volume (not neglected): $\mathcal{P}_{\rm n/piston} = \mathcal{P}_{F_{\rm n/piston}}$,
- a coupling between the powers $\mathcal{P}_{\rm c}$ (associated with the kinetic storage phenomenon), $\mathcal{P}_{F_{\rm p/piston}}$ and $\mathcal{P}_{F_{\rm n/piston}}$ through the conservation law expressing the Newton's second law ($\dot{p} = F_{\rm p/piston} - F_{\rm n/piston}$) applied to the piston and its mass (conservation with common flow –the mass velocity v–): $\mathcal{P}_{\rm c} = \mathcal{P}_{F_{\rm p/piston}} - \mathcal{P}_{F_{\rm n/piston}}$,
- a coupling between the powers $\mathcal{P}_{\rm p}$ (associated with the potential storage phenomenon in chamber p), $\mathcal{P}_{\rm p/piston}$ and $\mathcal{P}_{Q_{\rm p}}$ (associated with the chamber p ingoing fluid power) through the conservation law expressing the mass conservation ($\dot{q}_{\rm p} = Q_{\rm p} - \frac{{\rm d}V_{\rm p}}{{\rm d}t}$, conservation with common effort –pressure $P_{\rm p}$– with $Q_{\rm p}$ the chamber p ingoing volume flow rate): $\mathcal{P}_{\rm p} = \mathcal{P}_{Q_{\rm p}} - \mathcal{P}_{\rm p/piston}$,
- a coupling between the powers $\mathcal{P}_{\rm n}$ (associated with the potential storage phenomenon in chamber n), $\mathcal{P}_{\rm n/piston}$ and $\mathcal{P}_{Q_{\rm n}}$ (associated with the chamber n outgoing fluid power) through the conservation law expressing the mass conservation ($\dot{q}_{\rm n} = -Q_{\rm n} - \frac{{\rm d}V_{\rm n}}{{\rm d}t}$, conservation with common effort –pressure $P_{\rm n}$– with $Q_{\rm n}$ the chamber n outgoing volume flow rate): $\mathcal{P}_{\rm n} = -\mathcal{P}_{Q_{\rm n}} - \mathcal{P}_{\rm n/piston}$,
- a coupling between the powers $\mathcal{P}_{Q_{\rm p}}$, $\mathcal{P}_{Q_{\rm sp}}$ (associated with the hydraulic power at the output of the sp orifice) and $\mathcal{P}_{Q_{\rm pe}}$ (associated with the hydraulic power at the input of the pe orifice) through the conservation law expressing the mass conservation ($Q_{\rm p} = Q_{\rm sp} - Q_{\rm pe}$, conservation with common effort –pressure $P_{\rm p}$–): $\mathcal{P}_{Q_{\rm p}} = \mathcal{P}_{Q_{\rm sp}} - \mathcal{P}_{Q_{\rm pe}}$,
- a coupling between the powers $\mathcal{P}_{Q_{\rm n}}$, $\mathcal{P}_{Q_{\rm sn}}$ (associated with the hydraulic power at the output of the sn orifice) and $\mathcal{P}_{Q_{\rm ne}}$ (associated with the hydraulic power at the input of the ne orifice) through the conservation law expressing the mass conservation ($Q_{\rm n} = Q_{\rm sn} - Q_{\rm ne}$, conservation with common effort –pressure $P_{\rm n}$–): $\mathcal{P}_{Q_{\rm n}} = \mathcal{P}_{Q_{\rm sn}} - \mathcal{P}_{Q_{\rm ne}}$,
- a coupling between the powers $\mathcal{P}_{\rm sp}$, $\mathcal{P}_{Q_{\rm sp}}$ and $\mathcal{P}_{\rm S_{sp}}$ (associated with the hydraulic power of the fluid coming from the pressure source and going to the servo valve controlling the chamber p) through the conservation law $\Delta P_{\rm sp} = P_{\rm s} - P_{\rm p}$ (conservation at common flow –the volume flow rate $Q_{\rm sp}$–): $\mathcal{P}_{\rm sp} = \mathcal{P}_{\rm S_{sp}} - \mathcal{P}_{Q_{\rm sp}}$,

- a coupling between the powers $\mathcal{P}_{\mathrm{pe}}$, $\mathcal{P}_{Q_{\mathrm{pe}}}$ and $\mathcal{P}_{\mathrm{E_{pe}}}$ (associated with the hydraulic power of the fluid coming from the chamber p and going back to the tank) through the conservation law $\Delta P_{\mathrm{pe}} = P_{\mathrm{p}} - P_{\mathrm{e}}$ (conservation at common flow –the volume flow rate Q_{pe}–): $\mathcal{P}_{\mathrm{pe}} = \mathcal{P}_{\mathrm{E_{pe}}} - \mathcal{P}_{Q_{\mathrm{pe}}}$,

- a coupling between the powers $\mathcal{P}_{\mathrm{sn}}$, $\mathcal{P}_{Q_{\mathrm{sn}}}$ and $\mathcal{P}_{\mathrm{S_{sn}}}$ (associated with the hydraulic power of the fluid coming from the pressure source and going to the servo valve controlling the chamber n) through the conservation law $\Delta P_{\mathrm{sn}} = P_{\mathrm{s}} - P_{\mathrm{n}}$ (conservation at common flow –the volume flow rate Q_{sn}–): $\mathcal{P}_{\mathrm{sn}} = \mathcal{P}_{\mathrm{S_{sn}}} - \mathcal{P}_{Q_{\mathrm{sn}}}$,

- a coupling between the powers $\mathcal{P}_{\mathrm{ne}}$, $\mathcal{P}_{Q_{\mathrm{ne}}}$ and $\mathcal{P}_{\mathrm{E_{ne}}}$ (associated with the hydraulic power of the fluid coming from the chamber n and going back to the tank) through the conservation law $\Delta P_{\mathrm{ne}} = P_{\mathrm{n}} - P_{\mathrm{e}}$ (conservation at common flow –the volume flow rate Q_{ne}–): $\mathcal{P}_{\mathrm{ne}} = \mathcal{P}_{\mathrm{E_{ne}}} - \mathcal{P}_{Q_{\mathrm{ne}}}$,

- a coupling between the powers $\mathcal{P}_{\mathrm{s}}$ (associated with the pressure source), $\mathcal{P}_{\mathrm{S_{sp}}}$ and $\mathcal{P}_{\mathrm{S_{sn}}}$ through the conservation law expressing the mass conservation ($Q_{\mathrm{s}} = Q_{\mathrm{sp}} + Q_{\mathrm{sn}}$, conservation with common effort –pressure P_{s}–): $\mathcal{P}_{\mathrm{s}} = \mathcal{P}_{\mathrm{S_{sp}}} + \mathcal{P}_{\mathrm{S_{sn}}}$,

- a coupling between the powers $\mathcal{P}_{\mathrm{e}}$ (associated with the pressure in the tank), $\mathcal{P}_{\mathrm{S_{pe}}}$ and $\mathcal{P}_{\mathrm{S_{ne}}}$ through the conservation law expressing the mass conservation ($Q_{\mathrm{e}} = Q_{\mathrm{pe}} + Q_{\mathrm{ne}}$, conservation with common effort –pressure P_{e}–): $\mathcal{P}_{\mathrm{e}} = \mathcal{P}_{\mathrm{E_{pe}}} + \mathcal{P}_{\mathrm{E_{ne}}}$.

The sketch of the concept associations is given Fig. 5.

2 Structural analysis concepts

This section presents the concepts used in the sizing methodology and the physical structural analysis. They are expressed in terms of the basic concepts for modeling physical systems given in the previous section.

2.1 Definition

Structural analysis, from the physical point of view, is the study of a physical model properties independently of the numerical values of its parameters. One of the main benefits of structural analysis is to obtain characteristic properties true almost everywhere in the parameter space of the model.

2.2 Acausal concepts

The *acausal concepts* do not require the model variable assignment. They are essentially based on the existence of relations between powers in the model.

Definitions

Power line: A *power line* is a "way" along which energy flows between two points in a system. With respect to the basic concepts for modeling physical systems, a power line can be read through the energy structure and the multiport elementary physical phenomena (and the associated laws of conservation, of power conserving transduction, and of behavior). It is constituted of an ordered series of powers related by the different involved laws. By definition, a power of a power line must appear only once in the corresponding series.

Power line of energy supply: A *power line of energy supply* is a power line whose the first power is associated with an elementary physical phenomenon of a technological component that furnish energy to a system.

Modulation power line: A *modulation power line* is a power line whose the first power is involved in a component having an input modulation.

Input/Output (I/O) power line: A *I/O power line* is a power line between a power whose one of its power variables depends on a model input and, a power whose one of its power variables is related to an output.

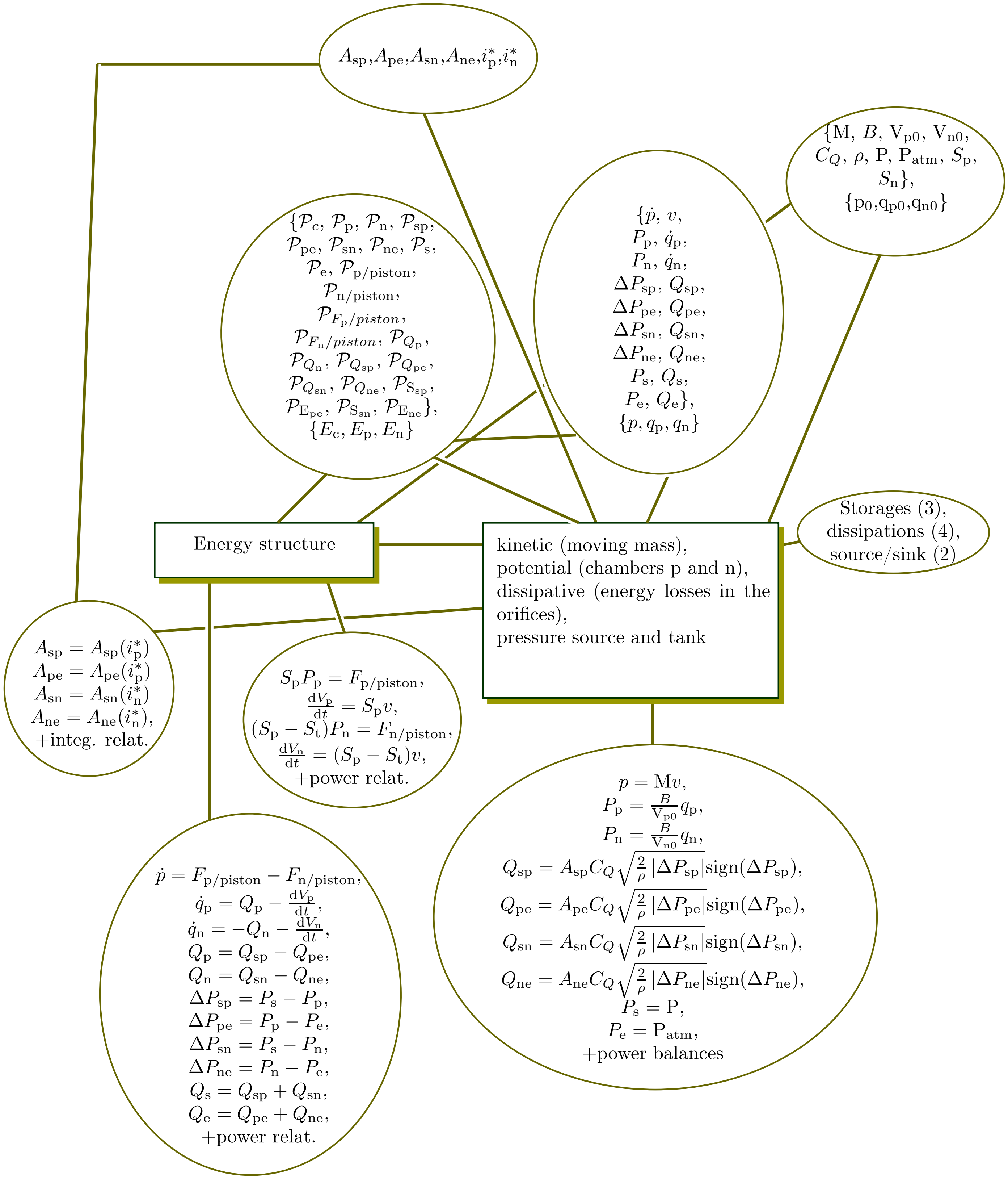

Figure 5: Concept associations for the hydraulic example

Joint/disjoint power lines: Two power lines are said *joint* if at least two powers, one of each line, are directly related by a conservation law. In the opposite case they are said *disjoint*. One necessary condition for two power lines to be disjoint is that they have no power in common.

Remark: In order to control a degree of freedom (from the mechanics point of view) and more generally an energetic degree of freedom (from the physics point of view), a necessary condition is the existence of a pair of power lines (one of energy supply and the other, a modulation power line) partially disjoint, that converge to the power associated with the elementary physical phenomenon related to this degree of freedom.

1st example: mechanical system

In Fig. 1 mechanical model, one of the power lines involves for instance the kinetic energy storage phenomenon with which the power $\mathcal{P}_c$ is associated, and the energy dissipation phenomenon with which the dissipation power $\mathcal{P}_d$ is associated. The relation between these powers is expressed by the conservation law in terms of powers: $\mathcal{P}_\mathrm{c} + \mathcal{P}_\mathrm{p} + \mathcal{P}_\mathrm{d} = 0$. A second power line is between the kinetic storage phenomenon ($\mathcal{P}_\mathrm{c}$) and the potential storage phenomenon ($\mathcal{P}_\mathrm{p}$) through the same coupling ($\mathcal{P}_\mathrm{c} + \mathcal{P}_\mathrm{p} + \mathcal{P}_\mathrm{d} = 0$). A third power line involves the potential energy storage phenomenon and the energy dissipation phenomenon ($\mathcal{P}_\mathrm{c} + \mathcal{P}_\mathrm{p} + \mathcal{P}_\mathrm{d} = 0$).

None of these power lines are of energy supply or modulation lines. Also all of them are joint.

2nd example : electrohydraulic system

Recall of the power relations (*cf.* Sec. 1.3)

The description of the energy structure enables, through the couplings, the relations between the powers in the model to be listed:

$$\begin{cases}
\boldsymbol{R}P1 & : \ \mathcal{P}_{\mathrm{p/piston}} = \mathcal{P}_{F_{\mathrm{p/piston}}} \\
\boldsymbol{R}P2 & : \ \mathcal{P}_{\mathrm{n/piston}} = \mathcal{P}_{F_{\mathrm{n/piston}}} \\
\boldsymbol{R}P3 & : \ \mathcal{P}_{\mathrm{c}} = \mathcal{P}_{F_{\mathrm{p/piston}}} - \mathcal{P}_{F_{\mathrm{n/piston}}} \\
\boldsymbol{R}P4 & : \ \mathcal{P}_{\mathrm{p}} = \mathcal{P}_{Q_\mathrm{p}} - \mathcal{P}_{\mathrm{p/piston}} \\
\boldsymbol{R}P5 & : \ \mathcal{P}_{\mathrm{n}} = -\mathcal{P}_{Q_\mathrm{n}} - \mathcal{P}_{\mathrm{n/piston}} \\
\boldsymbol{R}P6 & : \ \mathcal{P}_{Q_\mathrm{p}} = \mathcal{P}_{Q_\mathrm{sp}} - \mathcal{P}_{Q_\mathrm{pe}} \\
\boldsymbol{R}P7 & : \ \mathcal{P}_{Q_\mathrm{n}} = \mathcal{P}_{Q_\mathrm{sn}} - \mathcal{P}_{Q_\mathrm{ne}} \\
\boldsymbol{R}P8 & : \ \mathcal{P}_{\mathrm{sp}} = \mathcal{P}_{\mathrm{S_{sp}}} - \mathcal{P}_{Q_\mathrm{sp}} \\
\boldsymbol{R}P9 & : \ \mathcal{P}_{\mathrm{pe}} = \mathcal{P}_{\mathrm{E_{pe}}} - \mathcal{P}_{Q_\mathrm{pe}} \\
\boldsymbol{R}P10 & : \ \mathcal{P}_{\mathrm{sn}} = \mathcal{P}_{\mathrm{S_{sn}}} - \mathcal{P}_{Q_\mathrm{sn}} \\
\boldsymbol{R}P11 & : \ \mathcal{P}_{\mathrm{ne}} = \mathcal{P}_{\mathrm{E_{ne}}} - \mathcal{P}_{Q_\mathrm{ne}} \\
\boldsymbol{R}P12 & : \ \mathcal{P}_{\mathrm{s}} = \mathcal{P}_{\mathrm{S_{sp}}} + \mathcal{P}_{\mathrm{S_{sn}}} \\
\boldsymbol{R}P13 & : \ \mathcal{P}_{\mathrm{e}} = \mathcal{P}_{\mathrm{E_{pe}}} + \mathcal{P}_{\mathrm{E_{ne}}} \\
\boldsymbol{R}P14 & : \ \mathcal{P}_{\mathrm{c}} = \frac{\mathrm{d}E_\mathrm{c}}{\mathrm{d}t} \\
\boldsymbol{R}P15 & : \ \mathcal{P}_{\mathrm{p}} = \frac{\mathrm{d}E_\mathrm{p}}{\mathrm{d}t} \\
\boldsymbol{R}P16 & : \ \mathcal{P}_{\mathrm{n}} = \frac{\mathrm{d}E_\mathrm{n}}{\mathrm{d}t}
\end{cases}$$

Problem definition

To exhaustively list every power line in this model would be too much tedious and useless to illustrate the power line concept. This work is carried over to the search of the I/O power lines with respect to the set of inputs $\{u_1 = Q_\mathrm{sp}, u_2 = Q_\mathrm{sn}\}$ and the set of outputs $\{y_1 = v, y_2 = P_\mathrm{p}\}$. This choice corresponds, for instance, to the multivariable perspective of controlling, on one hand, the position of the mass moved by the cylinder and, on the other hand, the pressure in a chamber that enables, to some extent, the cylinder "stiffness" in a given position to be tuned. With respect to the basic physical concepts the chosen inputs and outputs are respectively associated with the phenomena of energy losses in the orifices "sp" and "sn" on one hand, and, with the kinetic phenomenon of the moving mass and the compressibility phenomenon in the chamber p on the other hand. Concerning the inputs, the power variables Q_sp and Q_sn are directly related to the modulating inputs i^*_p and i^*_n.

Acausal concepts of structural analysis

Related to the involved phenomena the previous inputs and outputs are associated with the powers $\mathcal{P}_{\rm sp}$ (for $Q_{\rm sp}$), $\mathcal{P}_{\rm sn}$ (for $Q_{\rm sn}$), $\mathcal{P}_{\rm c}$ (for v), et $\mathcal{P}_{\rm p}$ (for $P_{\rm p}$). With respect to these inputs and outputs there are, among others, four I/O power lines:

LP1 (between $\boldsymbol{Q}_{\rm sp}$ and $\boldsymbol{v}$): $\mathcal{P}_{\rm sp}$—$\mathcal{P}_{Q_{\rm sp}}$—$\mathcal{P}_{Q_{\rm p}}$—$\mathcal{P}_{\rm p/piston}$—$\mathcal{P}_{F_{\rm p/piston}}$—$\mathcal{P}_{\rm c}$. The involved relations are in order: RP8–RP6–RP4–RP1–RP3.

LP2 (between $\boldsymbol{Q}_{\rm sp}$ and $\boldsymbol{P}_{\rm p}$): $\mathcal{P}_{\rm sp}$—$\mathcal{P}_{Q_{\rm sp}}$—$\mathcal{P}_{Q_{\rm p}}$—$\mathcal{P}_{\rm p}$. The involved relations are in order: RP8–RP6–RP4.

LP3 (between $\boldsymbol{Q}_{\rm sn}$ and $\boldsymbol{v}$): $\mathcal{P}_{\rm sn}$—$\mathcal{P}_{Q_{\rm sn}}$—$\mathcal{P}_{Q_{\rm n}}$—$\mathcal{P}_{\rm n/piston}$—$\mathcal{P}_{F_{\rm n/piston}}$—$\mathcal{P}_{\rm c}$. The involved relations are in order: RP10–RP7–RP5–RP2–RP3.

LP4 (between $\boldsymbol{Q}_{\rm sn}$ and $\boldsymbol{P}_{\rm p}$): $\mathcal{P}_{\rm sn}$—$\mathcal{P}_{Q_{\rm sn}}$—$\mathcal{P}_{Q_{\rm n}}$—$\mathcal{P}_{\rm n/piston}$—$\mathcal{P}_{F_{\rm n/piston}}$—$\mathcal{P}_{F_{\rm p/piston}}$—$\mathcal{P}_{\rm p/piston}$—$\mathcal{P}_{\rm p}$. The involved relations are in order: RP10–RP7–RP5–RP2–RP3–RP1–RP4.

Among these power lines only LP2 and LP3 are disjoint and they are all modulation power line.

2.3 Causal concepts

Unlike the acausal concepts the *causal concepts* require the orientation of the model equations (and thus the variable assignments). The model equations are constituted of the behavior laws, the conservation laws, and the laws of energy transduction. This condition of equation orientation being supposed, the following definitions are introduced.

Definitions

Storage phenomenon port in integral causality: A port of a storage phenomenon in *integral causality* is a port of an elementary physical phenomenon (1-port or multiport) of energy storage type whose the associated coenergy variable is determined by a behavior law of the phenomenon.

Storage phenomenon port in derivative causality: A port of a storage phenomenon in *derivative causality* is a port of an elementary physical phenomenon (1-port or multiport) of energy storage type whose the associated energy variable is determined by a behavior law of the phenomenon.

Causal path: A *causal path* is an ordered series of variables related the ones to the others by the model equations without any one of them appearing more than once in the series.

I/O causal path: A *I/O causal path* is a causal path between an input and an output.

Disjoint causal paths: Two *disjoint causal paths* are two causal paths without any variable in common.

Length of a causal path: The *length of a causal path* is defined when the model equations are oriented in such a way that the higher number a possible of energy storage phenomena are in integral causality. Then the length is the number of energy storage phenomena in integral causality for the phenomena involved in the causal path.

Order of a causal path: The *order of a causal path* is defined when the model equations are oriented in such a way that the higher number as possible of energy storage phenomena are in integral causality. Then the order is the difference between the number of energy storage phenomena in integral causality on one hand, and, the number of energy storage phenomena in derivative causality on the other hand, for the phenomena involved in the causal path.

Output essential order: The *essential order* of an output is the higher number of time differentiation of this output necessary to express the inputs in an inverse model.

Remark:

- In a model the difference between the number of energy storage phenomena in integral causality and the one of energy storage phenomena in derivative causality defines the *model order*. This corresponds to the number of independent energy variables (constituting the state vector) and thus, to the number of independent initial conditions for these energy variables that are to be supplied to the model.
- The notion of causal path shows how a variable (mathematically) influences another algebraically, by the means of integrations (delays) or time differentiations (leads).

1$^{\text{st}}$ example: mechanical system

The oriented equations for this example are:

$$\begin{cases} \boldsymbol{E}Q1 & : \quad v = \frac{1}{\mathrm{m}}p \\ \boldsymbol{E}Q2 & : \quad F_{\mathrm{r}} = \mathrm{k}x \\ \boldsymbol{E}Q3 & : \quad F_{\mathrm{a}} = \mathrm{b}v \\ \boldsymbol{E}Q4 & : \quad \dot{p} = -F_{\mathrm{r}} - F_{\mathrm{a}} \\ \boldsymbol{E}Q5 & : \quad \dot{x} = v \end{cases}$$

A causal path between the variable x and the variable p is constituted of the series of variables x—F_{r}—$\dot{p}$—p through the equations EQ2–EQ4 in this order. Only one integration is involved in this causal path, thus, it is of length and of order 1.

2$^{\text{nd}}$ example : electrohydraulic system

The oriented equations for this example are:

$$\begin{cases} \boldsymbol{E}Q1 & : \quad P_{\mathrm{s}} = \mathrm{P} \\ \boldsymbol{E}Q2 & : \quad P_{\mathrm{e}} = \mathrm{P_{atm}} \\ \boldsymbol{E}Q3 & : \quad v = \frac{1}{\mathrm{M}}p \\ \boldsymbol{E}Q4 & : \quad P_{\mathrm{p}} = \frac{B}{\mathrm{V_{p0}}}q_{\mathrm{p}} \\ \boldsymbol{E}Q5 & : \quad P_{\mathrm{n}} = \frac{B}{\mathrm{V_{n0}}}q_{\mathrm{n}} \\ \boldsymbol{E}Q6 & : \quad \Delta P_{\mathrm{sp}} = P_{\mathrm{s}} - P_{\mathrm{p}} \\ \boldsymbol{E}Q7 & : \quad \Delta P_{\mathrm{pe}} = P_{\mathrm{p}} - P_{\mathrm{e}} \\ \boldsymbol{E}Q8 & : \quad \Delta P_{\mathrm{sn}} = P_{\mathrm{s}} - P_{\mathrm{n}} \\ \boldsymbol{E}Q9 & : \quad \Delta P_{\mathrm{ne}} = P_{\mathrm{n}} - P_{\mathrm{e}} \\ \boldsymbol{E}Q10 & : \quad Q_{\mathrm{sp}} = A_{\mathrm{sp}}(i_{\mathrm{p}}^*)C_Q\sqrt{\frac{2}{\rho}\,|\Delta P_{\mathrm{sp}}|}\mathrm{sign}(\Delta P_{\mathrm{sp}}) \\ \boldsymbol{E}Q11 & : \quad Q_{\mathrm{pe}} = A_{\mathrm{pe}}(i_{\mathrm{p}}^*)C_Q\sqrt{\frac{2}{\rho}\,|\Delta P_{\mathrm{pe}}|}\mathrm{sign}(\Delta P_{\mathrm{pe}}) \\ \boldsymbol{E}Q12 & : \quad Q_{\mathrm{sn}} = A_{\mathrm{sn}}(i_{\mathrm{n}}^*)C_Q\sqrt{\frac{2}{\rho}\,|\Delta P_{\mathrm{sn}}|}\mathrm{sign}(\Delta P_{\mathrm{sn}}) \\ \boldsymbol{E}Q13 & : \quad Q_{\mathrm{ne}} = A_{\mathrm{ne}}(i_{\mathrm{n}}^*)C_Q\sqrt{\frac{2}{\rho}\,|\Delta P_{\mathrm{ne}}|}\mathrm{sign}(\Delta P_{\mathrm{ne}}) \\ \boldsymbol{E}Q14 & : \quad Q_{\mathrm{p}} = Q_{\mathrm{sp}} - Q_{\mathrm{pe}} \\ \boldsymbol{E}Q15 & : \quad Q_{\mathrm{n}} = Q_{\mathrm{sn}} - Q_{\mathrm{ne}} \\ \boldsymbol{E}Q16 & : \quad \frac{\mathrm{d}V_{\mathrm{p}}}{\mathrm{d}t} = S_{\mathrm{p}}v \\ \boldsymbol{E}Q17 & : \quad \frac{\mathrm{d}V_{\mathrm{n}}}{\mathrm{d}t} = S_{\mathrm{n}}v \\ \boldsymbol{E}Q18 & : \quad \dot{q}_{\mathrm{p}} = Q_{\mathrm{p}} - \frac{\mathrm{d}V_{\mathrm{p}}}{\mathrm{d}t} \\ \boldsymbol{E}Q19 & : \quad \dot{q}_{\mathrm{n}} = -Q_{\mathrm{n}} - \frac{\mathrm{d}V_{\mathrm{n}}}{\mathrm{d}t} \\ \boldsymbol{E}Q20 & : \quad F_{\mathrm{p/piston}} = S_{\mathrm{p}}P_{\mathrm{p}} \\ \boldsymbol{E}Q21 & : \quad F_{\mathrm{n}/piston} = S_{\mathrm{n}}P_{\mathrm{n}} \\ \boldsymbol{E}Q22 & : \quad \dot{p} = F_{\mathrm{p/piston}} - F_{\mathrm{n/piston}} \\ \boldsymbol{E}Q23 & : \quad Q_{\mathrm{s}} = Q_{\mathrm{sp}} + Q_{\mathrm{sn}} \\ \boldsymbol{E}Q24 & : \quad Q_{\mathrm{e}} = Q_{\mathrm{pe}} + Q_{\mathrm{ne}} \\ \boldsymbol{E}Q25 & : \quad p = \int_0^t \dot{p}\mathrm{d}\tau + \mathrm{p_0} \\ \boldsymbol{E}Q26 & : \quad q_{\mathrm{p}} = \int_0^t \dot{q}_{\mathrm{p}}\mathrm{d}\tau + \mathrm{q_{p0}} \\ \boldsymbol{E}Q27 & : \quad q_{\mathrm{n}} = \int_0^t \dot{q}_{\mathrm{n}}\mathrm{d}\tau + \mathrm{q_{n0}} \end{cases}$$

It is worthwhile to note that it was possible in this system to put the port of each energy storage phenomenon in integral causality.

Without being exhaustive there are two disjoint causal paths between the set of inputs $\{u_1 = Q_{\mathrm{sp}}, u_2 = Q_{\mathrm{sn}}\}$ and the set of outputs $\{y_1 = v, y_2 = P_{\mathrm{p}}\}$:

CC1 (between $\boldsymbol{Q}_{\mathrm{sp}}$ and $\boldsymbol{P}_{\mathrm{p}}$): Q_{sp}—Q_{p}—$\dot{q}_{\mathrm{p}}$—q_{p}—P_{p} through the series of equations EQ14–EQ18–EQ26–EQ4,

CC2 (between $\boldsymbol{Q}_{\mathrm{sn}}$ and $\boldsymbol{v}$): Q_{sn}—Q_{n}—$\dot{q}_{\mathrm{n}}$—q_{n}—P_{n}—$F_{\mathrm{n}/piston}$—$\dot{p}$—p—v through the series of equations EQ15–EQ19–EQ27–EQ5–EQ21–EQ22–EQ25–EQ3.

It is simple to see here that the first path (CC1) is of length and of order 1 due to the potential storage phenomenon in chamber p in integral causality. The second path (CC2) is of length and of order 2 due to the two storage phenomena (potential in chamber n and kinetic) in integral causality.

3 Structural analysis criteria

This section presents the *criteria of structural analysis* implemented for testing model invertibility. They are expressed in terms of the concepts previously defined.

The analysis is carried out in three chronological steps corresponding to the criteria:

- of acausal structural invertibility: search of the sets of disjoint I/O power lines,
- of causal structural invertibility: search of the sets of disjoint I/O causal paths of minimal order,
- of the differentiability of the output specifications: this criteria uses the notion of output essential order.

The existence of the sets for the first two criteria determines the structural invertibility of the model and, in that sense, represents a necessary condition. If it is not fulfilled the model is not invertible. The sizing problem in consideration is then to be questioned. The third criterion checks the adequacy of the output specifications to the studied model structure.

3.1 Acausal criterion of structural invertibility

Criterion

The model is structurally not invertible if there is no set of disjoint I/O power lines.

This condition corresponds to the impossibility to simultaneously exchange the roles between the inputs and the outputs. A necessary and implicit condition is to deal with square model. However a square model is not necessarily structurally invertible. The existence of joint parts between power lines indicates the dependency between inputs or between outputs in the inverse model formulation.

Electrohydraulic example

In this example, with respect to the sets of inputs $\{u_1, u_2\}$ and outputs $\{y_1, y_2\}$, there are 4 power lines. Among these power lines, there is only one set of two disjoint I/O power lines (*cf.* Sec. 2.2). The first structural invertibility criterion is thus verified. The emphasized set of power lines, together with the causal paths identified in the application of the following criterion, will serve the orientation of the equations in order to obtain the inverse model. The following criteria give, on one hand, an adequacy condition between the output specifications and the model structure and, on the other hand, the constraints for the equation orientation in order to obtain the inverse model of minimal order.

3.2 Causal criterion of strutural invertibility

Criterion

The model is structurally not invertible if there is no set of disjoint I/O causal paths.

Remark:

- when there are several sets of disjoint I/O causal paths, the set whose the sum of the causal path lengths is minimal has to be chosen. This choice guaranty the minimal number of time differentiations of the outputs as well as a minimal order for the inverse model.

- the set of the disjoint I/O power lines associated with the set of disjoint I/O causal paths will guide the orientation of the equations in order to obtain the inverse model.

Electrohydraulic example

The results of Sec. 2.3 show that there exists one set of two disjoint I/O causal paths. The second criterion of structural invertibility is thus verified.

3.3 Differentiability criterion of the output specifications

Criterion

> Each output specification must mathematically be of class at least equal to the essential order of the corresponding output.

Remark:

The essential orders of the outputs are determined when the model equations are organized and oriented for determining the inverse model.

Electrohydraulic example

The results of Sec. 2.3 show that the output P_{p} specification must be a function a class C^1 and the output v specification must be a function of class C^2. On this example it would not be difficult to prove that the orders of the causal paths retained for the inversion correspond to the output essential orders.

4 Equation assignment constraints for an inverse model

With the application of the previous criteria the structural invertibility conditions are verified. At this stage the full invertibility of the model is not guarantied yet. In fact, it further depends on the mathematical properties of the relations touched by the inversion in the physical domains of the different model variables. Nevertheless the inversion process of the model may be started. Two constraints for the orientation of the equations are to be introduced. On one hand, the new variables playing the input and output roles must be indicated and, on the other hand, the equation inversion must be based on the I/O causal paths identified at the structural analysis step.

4.1 Constraints

1$^{\text{st}}$ contraint: inputs and outputs of the inverse model

> In the system of equations define the inputs as outputs and *vice versa*.

2$^{\text{nd}}$ contraint: inversion along the I/O causal paths

> Re-orient the equations involved in the I/O causal paths retained during the structural analysis step in order to determine, in the inverse sense, the different variables of each path from the output to the input.

Electrohydraulic example

For the electrohydraulic example the first constraint leads to define $\{u_{\text{inv1}} = v, u_{\text{inv2}} = P_{\text{p}}\}$ as the new inputs and, $\{y_{\text{inv1}} = Q_{\text{sp}}, y_{\text{inv2}} = Q_{\text{sn}}\}$ as the new outputs. The second constraint leads, for the causal path CC1, to re-orient the equations EQ14, EQ18, EQ26 and EQ4 in the following manner:

$$\begin{cases} \boldsymbol{E}Q4 & \longrightarrow & \boldsymbol{E}Q4_{\text{inv}} & : & q_{\text{p}} = \frac{\text{V}_{\text{p0}}}{B} P_{\text{p}} \\ \boldsymbol{E}Q26 & \longrightarrow & \boldsymbol{E}Q26_{\text{inv}} & : & \dot{q}_{\text{p}} = \frac{\text{d}q_{\text{p}}}{\text{d}t} \\ \boldsymbol{E}Q18 & \longrightarrow & \boldsymbol{E}Q18_{\text{inv}} & : & Q_{\text{p}} = \dot{q}_{\text{p}} + \frac{\text{d}V_{\text{p}}}{\text{d}t} \\ \boldsymbol{E}Q14 & \longrightarrow & \boldsymbol{E}Q14_{\text{inv}} & : & Q_{\text{sp}} = Q_{\text{p}} + Q_{\text{pe}} \end{cases}$$

For the causal path CC2 the equations EQ15, EQ19, EQ27, EQ5, EQ21, EQ22, EQ25 and EQ3 are re-oriented in the following manner:

$$\begin{cases} \boldsymbol{E}Q3 & \longrightarrow & \boldsymbol{E}Q3_{\text{inv}} & : & p = \text{M}v \\ \boldsymbol{E}Q25 & \longrightarrow & \boldsymbol{E}Q25_{\text{inv}} & : & \dot{p} = \frac{\text{d}p}{\text{d}t} \\ \boldsymbol{E}Q22 & \longrightarrow & \boldsymbol{E}Q22_{\text{inv}} & : & F_{\text{n/piston}} = F_{\text{p/piston}} - \dot{p} \\ \boldsymbol{E}Q21 & \longrightarrow & \boldsymbol{E}Q21_{\text{inv}} & : & P_{\text{n}} = \frac{1}{S_{\text{n}}} F_{\text{n}/piston} \\ \boldsymbol{E}Q5 & \longrightarrow & \boldsymbol{E}Q5_{\text{inv}} & : & q_{\text{n}} = \frac{\text{V}_{\text{n0}}}{B} P_{\text{n}} \\ \boldsymbol{E}Q27 & \longrightarrow & \boldsymbol{E}Q27_{\text{inv}} & : & \dot{q}_{\text{n}} = \frac{\text{d}q_{\text{n}}}{\text{d}t} \\ \boldsymbol{E}Q19 & \longrightarrow & \boldsymbol{E}Q19_{\text{inv}} & : & Q_{\text{n}} = -\dot{q}_{\text{n}} - \frac{\text{d}V_{\text{n}}}{\text{d}t} \\ \boldsymbol{E}Q15 & \longrightarrow & \boldsymbol{E}Q15_{\text{inv}} & : & Q_{\text{sn}} = Q_{\text{n}} + Q_{\text{ne}} \end{cases}$$

Remark:

According to structural analysis it appears one time differentiation of the initial model output $y_2 = u_{\text{inv2}} = P_{\text{p}}$ in the equation EQ26_{inv} (in the causal path CC1) and, two time differentiations of the output $y_1 = u_{\text{inv1}} = v$ in the equations EQ25_{inv} and EQ27_{inv} (in the causal path CC2).

5 Definition of the data model

5.1 Data associated with the basic concepts for modeling physical systems

The implementation of the sizing methodology requires the definition of a data model. This data model is based on the basic physical concepts and the associations between them as presented in Fig. 2 diagram. The data of this model are listed in Tab. 3 for the concepts, and Tab. 4 gives the axioms on these data for the associations between the concepts.

Definition of the port number (*cf.* Sec. 1.1) :

In addition to the data associated with the basic physical concepts, a positive integer represents their *port number*. This port number is defined as the number of data P (resp. the number of data P-1) for the concepts ① (resp. for the concepts ⑤ with which they are associated (associations ①⟷② –resp. ②⟷⑤–).

The associations presented in the axioms ①⟷① and ①⟷⑤ bring the corresponding data P in the associations expressed in the axioms ①⟷② and ②⟷⑤. This means that the same piece of data P can be associated with more than one piece of data of the concepts ① and ⑤. On the contrary the following postulate is given: a same piece of data P cannot be associated with more than 2 data of the concepts ① and ⑤. This postulate will be of first importance in the specifications of the graphic editor.

Table 3: Table of data associated with the basic concepts for physical system modeling.

Concepts	Data[1]	Constraints	Comments
Concept 1 Energy structure	n_{J1} J1 : $\{J1_1,\cdots,J1_{n_{J1}}\}$		Energy conservation elements
	n_{J0} J0 : $\{J0_1,\cdots,J0_{n_{J0}}\}$		
	n_{TF} TF : $\{TF_1,\cdots,TF_{n_{TF}}\}$		Power conserving elements of energy transduction
	n_{GY} GY : $\{GY_1,\cdots,GY_{n_{GY}}\}$		
Concept 2 Power, energy	n_P P : $\{P_1,\cdots,P_{n_P}\}$		Power
	n_p EN : $\{EN_1,\cdots,EN_{n_p}\}$		Energy
Concept 3 Conservation laws	n_{J1} LCB1 : $\{LCB1_1,\cdots,LCB1_{n_{J1}}\}$		Effort balance.
	n_{J0} LCB0 : $\{LCB0_1,\cdots,LCB0_{n_{J0}}\}$		Flow balance
	n_{LCE1} LCE1 : $\{LCE1_1,\cdots,LCE1_{n_{LCE1}}\}$		Flow equality
	n_{LCE0} LCE0 : $\{LCE0_1,\cdots,LCE0_{n_{LCE0}}\}$		Effort equality
	$n_{J1}+n_{J0}$ BP : $\{BP_1,\cdots,BP_{n_{J1}},BP_{n_{J1}+1},\cdots,BP_{n_{J1}+n_{J0}}\}$		Power balance
Concept 4 Laws of power conserving energy transduction	$2(n_{TF}+n_{GY})$ LTE : $\{LTE_1,\cdots,LTE_{2n_{TF}},LTE_{2n_{TF}+1},\cdots,LTE_{2(n_{TF}+n_{GY})}\}$		Laws of power conserving energy transduction
	$n_{TF}+n_{GY}$ EP : $\{EP_1,\cdots,EP_{n_{TF}},EP_{n_{TF}+1},\cdots,EP_{n_{TF}+n_{GY}}\}$		Power equalities
Concept 5 Elementary physical phenomena	n_{PPE} PPE : $\{PPE_1,\cdots,PPE_{n_{PPE}}\}$		Elementary physical phenomena
Concept 6 Phenomenon type	7 TYPE : {I,C,IC,R,SE,SF,free}		Types of energy phenomena
Concept 7 Behavior laws	n_{LC} LC : $\{LC_1,\cdots,LC_{n_{LC}}\}$		Behavior laws
	n_{PPE} LCBP : $\{LCBP_1,\cdots,LCBP_{n_{PPE}}\}$		Power balance
Concept 8 Power and energy variables	n_{VP} E : $\{E_1,\cdots,E_{n_{VP}}\}$	$0\leqslant n_{VP}\leqslant n_P$	Effort variables
	n_{VP} F : $\{F_1,\cdots,F_{n_{VP}}\}$		Flow variables
	n_M M : $\{M_1,\cdots,M_{n_M}\}$	$0\leqslant n_M+n_D\leqslant n_{VP}$	Generalized momenta
	n_D D : $\{D_1,\cdots,D_{n_D}\}$		Generalized displacements
Concept 9 Supplementary variables	n_{MOD} MOD : $\{MOD_1,\cdots,MOD_{n_{MOD}}\}$		Modulation variables
	n_{IN} IN : $\{IN_1,\cdots,IN_{n_{IN}}\}$		Input variables
	n_{OUT} OUT : $\{OUT_1,\cdots,OUT_{n_{OUT}}\}$		Output variables
	n_{CAL} CAL : $\{CAL_1,\cdots,CAL_{n_{CAL}}\}$		Intermediary calculus variables
Concept 10 Par., Init. cond.	n_{PAR} PAR : $\{PAR_1,\cdots,PAR_{n_{PAR}}\}$		Parameters
	n_{CI} CI : $\{CI_1,\cdots,CI_{n_{CI}}\}$		Initial conditions
Concept 11 Calculus relations	n_{INT} INT : $\{INT_1,\cdots,INT_{n_{INT}}\}$		Integrators
	n_{DIF} DIF : $\{DIF_1,\cdots,DIF_{n_{DIF}}\}$		Time differentiators.
	n_{ALG} ALG : $\{ALG_1,\cdots,ALG_{n_{ALG}}\}$		Algebraic relations

[1] The quantities n_{XXXX} are integers.

Table 4: Axioms on the model data for the associations between the basic physical concepts.

Associat.	Axioms	Constraints	Comments
①⟷①	$\forall i \in [1, \mathrm{n_{J1}}]$, $\exists$ $\mathrm{n}_{\mathrm{J1J1}i}$ J1$\neq$J1$_i$ associated with J1$_i$ $\exists$ $\mathrm{n}_{\mathrm{J1J0}i}$ J0 associated with J1$_i$ $\exists$ $\mathrm{n}_{\mathrm{J1TF}i}$ TF associated with J1$_i$ $\exists$ $\mathrm{n}_{\mathrm{J1GY}i}$ GY associated with J1$_i$	$0\leqslant \mathrm{n}_{\mathrm{J1J1}i} \leqslant \mathrm{n}_{\mathrm{J1}i}$ $0\leqslant \mathrm{n}_{\mathrm{J1J0}i} \leqslant \mathrm{n}_{\mathrm{J1}i}$ $0\leqslant \mathrm{n}_{\mathrm{J1TF}i} \leqslant \mathrm{n}_{\mathrm{J1}i}$ $0\leqslant \mathrm{n}_{\mathrm{J1GY}i} \leqslant \mathrm{n}_{\mathrm{J1}i}$	$\mathrm{n}_{\mathrm{J1}i}$ defined below
	$\forall i \in [1, \mathrm{n_{J0}}]$, $\exists$ $\mathrm{n}_{\mathrm{J0J1}i}$ J1 associated with J0$_i$ $\exists$ $\mathrm{n}_{\mathrm{J0J0}i}$ J0$\neq$J0$_i$ associated with J0$_i$ $\exists$ $\mathrm{n}_{\mathrm{J0TF}i}$ TF associated with J0$_i$ $\exists$ $\mathrm{n}_{\mathrm{J0GY}i}$ GY associated with J0$_i$	$0\leqslant \mathrm{n}_{\mathrm{J0J1}i} \leqslant \mathrm{n}_{\mathrm{J0}i}$ $0\leqslant \mathrm{n}_{\mathrm{J0J0}i} \leqslant \mathrm{n}_{\mathrm{J0}i}$ $0\leqslant \mathrm{n}_{\mathrm{J0TF}i} \leqslant \mathrm{n}_{\mathrm{J0}i}$ $0\leqslant \mathrm{n}_{\mathrm{J0GY}i} \leqslant \mathrm{n}_{\mathrm{J0}i}$	$\mathrm{n}_{\mathrm{J0}i}$ defined below
	$\forall i \in [1, \mathrm{n_{TF}}]$, $\exists$ $\mathrm{n}_{\mathrm{TFJ1}i}$ J1 associated with TF$_i$ $\exists$ $\mathrm{n}_{\mathrm{TFJ0}i}$ J0 associated with TF$_i$ $\exists$ $\mathrm{n}_{\mathrm{TFTF}i}$ TF$\neq$TF$_i$ associated with TF$_i$ $\exists$ $\mathrm{n}_{\mathrm{TFGY}i}$ GY associated with TF$_i$	$0\leqslant \mathrm{n}_{\mathrm{TFJ1}i} \leqslant 2$ $0\leqslant \mathrm{n}_{\mathrm{TFJ0}i} \leqslant 2$ $0\leqslant \mathrm{n}_{\mathrm{TFTF}i} \leqslant 2$ $0\leqslant \mathrm{n}_{\mathrm{TFGY}i} \leqslant 2$	
	$\forall i \in [1, \mathrm{n_{GY}}]$, $\exists$ $\mathrm{n}_{\mathrm{GYJ1}i}$ J1 associated with GY$_i$ $\exists$ $\mathrm{n}_{\mathrm{GYJ0}i}$ J0 associated with GY$_i$ $\exists$ $\mathrm{n}_{\mathrm{GYTF}i}$ TF associated with GY$_i$ $\exists$ $\mathrm{n}_{\mathrm{GYGY}i}$ GY$\neq$GY$_i$ associated with GY$_i$	$0\leqslant \mathrm{n}_{\mathrm{GYJ1}i} \leqslant 2$ $0\leqslant \mathrm{n}_{\mathrm{GYJ0}i} \leqslant 2$ $0\leqslant \mathrm{n}_{\mathrm{GYTF}i} \leqslant 2$ $0\leqslant \mathrm{n}_{\mathrm{GYGY}i} \leqslant 2$	
①⟷②	$\forall i \in [1, \mathrm{n_{J1}}]$, $\exists$ $\mathrm{n}_{\mathrm{J1}i}$ P associated with J1$_i$	$1\leqslant \mathrm{n}_{\mathrm{J1}i} \leqslant \mathrm{n_P}$	$\mathrm{n}_{\mathrm{J1}i}$ ports
	$\forall i \in [1, \mathrm{n_{J0}}]$, $\exists$ $\mathrm{n}_{\mathrm{J0}i}$ P associated with J0$_i$	$1\leqslant \mathrm{n}_{\mathrm{J0}i} \leqslant \mathrm{n_P}$	$\mathrm{n}_{\mathrm{J0}i}$ ports
	$\forall i \in [1, \mathrm{n_{TF}}]$, $\exists$ 2 P associated with TF$_i$		2 ports
	$\forall i \in [1, \mathrm{n_{GY}}]$, $\exists$ 2 P associated with GY$_i$		2 ports
①⟷③	$\forall i \in [1, \mathrm{n_{J1}}]$, J1$_i$, LCB1$_i$ and BP$_i$ are associated, and $\exists$ $\mathrm{n}_{\mathrm{LCE1}i}$ LCE1 associated with J1$_i$	$1\leqslant \mathrm{n}_{\mathrm{LCE1}i} = \mathrm{n}_{\mathrm{J1}i} - 1$ $< \mathrm{n_{LCE1}} \leqslant \mathrm{n_P}$	*Cf.* rem. 1 to 3
	$\forall i \in [1, \mathrm{n_{J0}}]$, J0$_i$, LCB0$_i$ and BP$_{\mathrm{n_{J1}}+i}$ are associated, and $\exists$ $\mathrm{n}_{\mathrm{LCE0}i}$ LCE0 associated with J0$_i$	$1\leqslant \mathrm{n}_{\mathrm{LCE0}i} = \mathrm{n}_{\mathrm{J0}i} - 1$ $< \mathrm{n_{LCE0}} \leqslant \mathrm{n_P}$	*Cf.* rem. 4 to 6
①⟷④	$\forall i \in [1, \mathrm{n_{TF}}]$, LTE$_{2i-1}$, LTE$_{2i}$ and EP$_i$ are associated with TF$_i$		*Cf.* rem. 7 to 9
	$\forall i \in [1, \mathrm{n_{GY}}]$, LTE$_{2\mathrm{n_{TF}}+2i-1}$, LTE$_{2\mathrm{n_{TF}}+2i}$ and EP$_{\mathrm{n_{TF}}+i}$ are associated with GY$_i$		*Cf.* rem. 10 to 12
①⟷⑤	$\forall i \in [1, \mathrm{n_{J1}}]$, $\exists$ $\mathrm{n}_{\mathrm{J1PPE}i}$ PPE associated with J1$_i$	$0\leqslant \mathrm{n}_{\mathrm{J1PPE}i} \leqslant \mathrm{n}_{\mathrm{J1}i}$ $\mathrm{n}_{\mathrm{J1J1}i}+\mathrm{n}_{\mathrm{J1J0}i}+\mathrm{n}_{\mathrm{J1TF}i}+\mathrm{n}_{\mathrm{J1GY}i}$ $+\mathrm{n}_{\mathrm{J1PPE}i}=\mathrm{n}_{\mathrm{J1}i}$	
	$\forall i \in [1, \mathrm{n_{J0}}]$, $\exists$ $\mathrm{n}_{\mathrm{J0PPE}i}$ PPE associated with J0$_i$	$0\leqslant \mathrm{n}_{\mathrm{J0PPE}i} \leqslant \mathrm{n}_{\mathrm{J0}i}$ $\mathrm{n}_{\mathrm{J0J1}i}+\mathrm{n}_{\mathrm{J0J0}i}+\mathrm{n}_{\mathrm{J0TF}i}+\mathrm{n}_{\mathrm{J0GY}i}$ $+\mathrm{n}_{\mathrm{J0PPE}i}=\mathrm{n}_{\mathrm{J0}i}$	
	$\forall i \in [1, \mathrm{n_{TF}}]$, $\exists$ $\mathrm{n}_{\mathrm{TFPPE}i}$ PPE associated with TF$_i$	$0\leqslant \mathrm{n}_{\mathrm{TFPPE}i} \leqslant 2$ $\mathrm{n}_{\mathrm{TFJ1}i}+\mathrm{n}_{\mathrm{TFJ0}i}+\mathrm{n}_{\mathrm{TFTF}i}+\mathrm{n}_{\mathrm{TFGY}i}$ $+\mathrm{n}_{\mathrm{TFPPE}i}=2$	
	$\forall i \in [1, \mathrm{n_{GY}}]$, $\exists$ $\mathrm{n}_{\mathrm{GYPPE}i}$ PPE associated with GY$_i$	$0\leqslant \mathrm{n}_{\mathrm{GYPPE}i} \leqslant 2$ $\mathrm{n}_{\mathrm{GYJ1}i}+\mathrm{n}_{\mathrm{GYJ0}i}+\mathrm{n}_{\mathrm{GYTF}i}+\mathrm{n}_{\mathrm{GYGY}i}$ $+\mathrm{n}_{\mathrm{GYPPE}i}=2$	
①⟷⑧	$\forall i \in [1, \mathrm{n_{J1}}]$, $\exists$ $\mathrm{n}_{\mathrm{J1}i}$ E and $\mathrm{n}_{\mathrm{J1}i}$ F associated with J1$_i$		*Cf.* rem. 13
	$\forall i \in [1, \mathrm{n_{J0}}]$, $\exists$ $\mathrm{n}_{\mathrm{J0}i}$ E and $\mathrm{n}_{\mathrm{J0}i}$ F associated with J0$_i$		
	$\forall i \in [1, \mathrm{n_{TF}}]$, $\exists$ 2 E and 2 F associated with TF$_i$		
	$\forall i \in [1, \mathrm{n_{GY}}]$, $\exists$ 2 E and 2 F associated with GY$_i$		
①⟷⑨	$\forall i \in [1, \mathrm{n_{TF}}]$, $\exists$ $\mathrm{n}_{\mathrm{TFMOD}i}$ MOD associated with TF$_i$	$0\leqslant \mathrm{n}_{\mathrm{TFMOD}i} \leqslant 1$	Modulations
	$\forall i \in [1, \mathrm{n_{GY}}]$, $\exists$ $\mathrm{n}_{\mathrm{GYMOD}i}$ MOD associated with GY$_i$	$0\leqslant \mathrm{n}_{\mathrm{GYMOD}i} \leqslant 1$	Modulations

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Associat.	Axioms	Constraints	Comments
$1 \longleftrightarrow 10$	$\forall i \in [1, \mathrm{n_{TF}}]$, $\exists$ $\mathrm{n_{TFPAR}}_i$ PAR associated with TF_i	$0 \leqslant \mathrm{n_{TFPAR}}_i \leqslant \mathrm{n_{PAR}}$	
	$\forall i \in [1, \mathrm{n_{GY}}]$, $\exists$ $\mathrm{n_{GYPAR}}_i$ PAR associated with GY_i	$0 \leqslant \mathrm{n_{GYPAR}}_i \leqslant \mathrm{n_{PAR}}$	
$2 \longleftrightarrow 2$	$\forall$ P, $\exists$ 1 EN associated with P		*Cf.* rem. 14
$2 \longleftrightarrow 5$	$\forall i \in [1, \mathrm{n_{PPE}}]$, $\exists$ $\mathrm{n_{PPE}}_i$+1 P and $\mathrm{n_{PPE}}_i$+1 EN associated with PPE_i	$2 \leqslant \mathrm{n_{PPE}}_i + 1 \leqslant \mathrm{n_P}$ $\mathrm{n_{LC}} = \sum_{i=1}^{\mathrm{n_{PPE}}} \mathrm{n_{PPE}}_i$	$\mathrm{n_{PPE}}_i$ ports *Cf.* rem. 15
$2 \longleftrightarrow 8$	$\forall i \in [1, \mathrm{n_{VP}}]$, $\exists$ 1 P associated with E_i and F_i		P=E×F
	$\forall i \in [1, \mathrm{n_M}]$, $\exists$ 1 EN associated with M_i		*Cf.* rem. 16
	$\forall i \in [1, \mathrm{n_D}]$, $\exists$ 1 EN associated with D_i		
$2 \longleftrightarrow 11$	$\forall i \in [1, \mathrm{n_P}]$, $\exists$ $\mathrm{n_{PINT}}_i$ INT associated with P_i	$0 \leqslant \mathrm{n_{PINT}}_i \leqslant \mathrm{n_{INT}}$	
	$\forall i \in [1, \mathrm{n_P}]$, $\exists$ $\mathrm{n_{PDIF}}_i$ DIF associated with P_i	$0 \leqslant \mathrm{n_{PDIF}}_i \leqslant \mathrm{n_{DIF}}$	
	$\forall i \in [1, \mathrm{n_P}]$, $\exists$ $\mathrm{n_{PALG}}_i$ ALG associated with P_i	$0 \leqslant \mathrm{n_{PALG}}_i \leqslant \mathrm{n_{ALG}}$	
	$\forall i \in [1, \mathrm{n_P}]$, $\exists$ $\mathrm{n_{EINT}}_i$ INT associated with E_i	$0 \leqslant \mathrm{n_{EINT}}_i \leqslant \mathrm{n_{INT}}$	
	$\forall i \in [1, \mathrm{n_P}]$, $\exists$ $\mathrm{n_{EDIF}}_i$ DIF associated with E_i	$0 \leqslant \mathrm{n_{EDIF}}_i \leqslant \mathrm{n_{DIF}}$	
	$\forall i \in [1, \mathrm{n_P}]$, $\exists$ $\mathrm{n_{EALG}}_i$ ALG associated with E_i	$0 \leqslant \mathrm{n_{EALG}}_i \leqslant \mathrm{n_{ALG}}$	
$5 \longleftrightarrow 6$	$\forall i \in [1, \mathrm{n_{PPE}}]$, $\exists$ 1 TYPE associated with PPE_i		
$5 \longleftrightarrow 7$	$\forall i \in [1, \mathrm{n_{PPE}}]$, $\mathrm{n_{PPE}}_i$ LC and 1 LCBP are associated with PPE_i		*Cf.* rem. 17 and 18
$5 \longleftrightarrow 8$	$\forall i \in [1, \mathrm{n_M}]$, $\exists$ 1 PPE associated with M_i		
	$\forall i \in [1, \mathrm{n_D}]$, $\exists$ 1 PPE associated with D_i		
$5 \longleftrightarrow 9$	$\forall i \in [1, \mathrm{n_{PPE}}]$, $\exists$ $\mathrm{n_{PPEMOD}}_i$ MOD associated with PPE_i	$0 \leqslant \mathrm{n_{PPEMOD}}_i \leqslant 1$	*Cf.* rem. 19
	$\forall i \in [1, \mathrm{n_{PPE}}]$, $\exists$ $\mathrm{n_{PPECAL}}_i$ CAL associated with PPE_i	$0 \leqslant \mathrm{n_{PPECAL}}_i \leqslant \mathrm{n_{CAL}}$	
$5 \longleftrightarrow 10$	$\forall i \in [1, \mathrm{n_{PPE}}]$, $\exists$ $\mathrm{n_{PPEPAR}}_i$ PAR associated with PPE_i	$0 \leqslant \mathrm{n_{PPEPAR}}_i \leqslant \mathrm{n_{PAR}}$	
	$\forall i \in [1, \mathrm{n_{PPE}}]$, $\exists$ $\mathrm{n_{PPECI}}_i$ CI associated with PPE_i	$\mathrm{n_{PPECI}}_i$ =0 or $\mathrm{n_{PPE}}_i$	
$5 \longleftrightarrow 11$	$\forall i \in [1, \mathrm{n_{PPE}}]$, $\exists$ $\mathrm{n_{PPEINT}}_i$ INT associated with PPE_i	$\mathrm{n_{PPEINT}}_i = 0$ or $\mathrm{n_{PPE}}_i$	*Cf.* rem. 20
	$\forall i \in [1, \mathrm{n_{PPE}}]$, $\exists$ $\mathrm{n_{PPEDIF}}_i$ DIF associated with PPE_i	$0 \leqslant \mathrm{n_{PPEDIF}}_i \leqslant \mathrm{n_{DIF}}$	
	$\forall i \in [1, \mathrm{n_{PPE}}]$, $\exists$ $\mathrm{n_{PPEALG}}_i$ ALG associated with PPE_i	$0 \leqslant \mathrm{n_{PPEALG}}_i \leqslant \mathrm{n_{ALG}}$	
$8 \longleftrightarrow 8$	$\forall i \in [1, \mathrm{n_M}]$, $\exists$ 1 E associated with M_i		$\mathrm{E}=\frac{\mathrm{dM}}{\mathrm{d}t}$
	$\forall i \in [1, \mathrm{n_D}]$, $\exists$ 1 F associated with D_i		$\mathrm{F}=\frac{\mathrm{dD}}{\mathrm{d}t}$
$8 \longleftrightarrow 11$	$\forall i \in [1, \mathrm{n_{VP}}]$, $\exists$ $\mathrm{n_{EINT}}_i$ INT associated with E_i	$0 \leqslant \mathrm{n_{EINT}}_i \leqslant \mathrm{n_{INT}}$	
	$\forall i \in [1, \mathrm{n_{VP}}]$, $\exists$ $\mathrm{n_{FINT}}_i$ INT associated with F_i	$0 \leqslant \mathrm{n_{FINT}}_i \leqslant \mathrm{n_{INT}}$	
	$\forall i \in [1, \mathrm{n_M}]$, $\exists$ $\mathrm{n_{MINT}}_i$ INT associated with M_i	$0 \leqslant \mathrm{n_{MINT}}_i \leqslant \mathrm{n_{INT}}$	
	$\forall i \in [1, \mathrm{n_D}]$, $\exists$ $\mathrm{n_{DINT}}_i$ INT associated with D_i	$0 \leqslant \mathrm{n_{DINT}}_i \leqslant \mathrm{n_{INT}}$	
	$\forall i \in [1, \mathrm{n_{VP}}]$, $\exists$ $\mathrm{n_{EDIF}}_i$ DIF associated with E_i	$0 \leqslant \mathrm{n_{EDIF}}_i \leqslant \mathrm{n_{DIF}}$	
	$\forall i \in [1, \mathrm{n_{VP}}]$, $\exists$ $\mathrm{n_{FDIF}}_i$ DIF associated with F_i	$0 \leqslant \mathrm{n_{FDIF}}_i \leqslant \mathrm{n_{DIF}}$	
	$\forall i \in [1, \mathrm{n_M}]$, $\exists$ $\mathrm{n_{MDIF}}_i$ DIF associated with M_i	$0 \leqslant \mathrm{n_{MDIF}}_i \leqslant \mathrm{n_{DIF}}$	
	$\forall i \in [1, \mathrm{n_D}]$, $\exists$ $\mathrm{n_{DDIF}}_i$ DIF associated with D_i	$0 \leqslant \mathrm{n_{DDIF}}_i \leqslant \mathrm{n_{DIF}}$	
	$\forall i \in [1, \mathrm{n_{VP}}]$, $\exists$ $\mathrm{n_{EALG}}_i$ ALG associated with E_i	$0 \leqslant \mathrm{n_{EALG}}_i \leqslant \mathrm{n_{ALG}}$	
	$\forall i \in [1, \mathrm{n_{VP}}]$, $\exists$ $\mathrm{n_{FALG}}_i$ ALG associated with F_i	$0 \leqslant \mathrm{n_{FALG}}_i \leqslant \mathrm{n_{ALG}}$	
	$\forall i \in [1, \mathrm{n_M}]$, $\exists$ $\mathrm{n_{MALG}}_i$ ALG associated with M_i	$0 \leqslant \mathrm{n_{MALG}}_i \leqslant \mathrm{n_{ALG}}$	
	$\forall i \in [1, \mathrm{n_D}]$, $\exists$ $\mathrm{n_{DALG}}_i$ ALG associated with D_i	$0 \leqslant \mathrm{n_{DALG}}_i \leqslant \mathrm{n_{ALG}}$	
$9 \longleftrightarrow 11$	$\forall i \in [1, \mathrm{n_{MOD}}]$, $\exists$ $\mathrm{n_{MODINT}}_i$ INT associated with MOD_i	$0 \leqslant \mathrm{n_{MODINT}}_i \leqslant 1$	
	$\forall i \in [1, \mathrm{n_{MOD}}]$, $\exists$ $\mathrm{n_{MODDIF}}_i$ DIF associated with MOD_i	$0 \leqslant \mathrm{n_{MODDIF}}_i \leqslant 1$	
	$\forall i \in [1, \mathrm{n_{MOD}}]$, $\exists$ $\mathrm{n_{MODALG}}_i$ ALG associated with MOD_i	$0 \leqslant \mathrm{n_{MODALG}}_i \leqslant 2$	
	$\forall i \in [1, \mathrm{n_{IN}}]$, $\exists$ $\mathrm{n_{ININT}}_i$ INT associated with IN_i	$0 \leqslant \mathrm{n_{ININT}}_i \leqslant 1$	
	$\forall i \in [1, \mathrm{n_{IN}}]$, $\exists$ $\mathrm{n_{INDIF}}_i$ DIF associated with IN_i	$0 \leqslant \mathrm{n_{INDIF}}_i \leqslant 1$	*Cf.* rem. 21
	$\forall i \in [1, \mathrm{n_{IN}}]$, $\exists$ $\mathrm{n_{INALG}}_i$ ALG associated with IN_i	$0 \leqslant \mathrm{n_{INALG}}_i \leqslant 1$	
	$\forall i \in [1, \mathrm{n_{OUT}}]$, $\exists$ $\mathrm{n_{OUTINT}}_i$ INT associated with OUT_i	$0 \leqslant \mathrm{n_{OUTINT}}_i \leqslant 1$	
	$\forall i \in [1, \mathrm{n_{OUT}}]$, $\exists$ $\mathrm{n_{OUTDIF}}_i$ DIF associated with OUT_i	$0 \leqslant \mathrm{n_{OUTDIF}}_i \leqslant 1$	*Cf.* rem. 22
	$\forall i \in [1, \mathrm{n_{OUT}}]$, $\exists$ $\mathrm{n_{OUTALG}}_i$ ALG associated with OUT_i	$0 \leqslant \mathrm{n_{OUTALG}}_i \leqslant 1$	
	$\forall i \in [1, \mathrm{n_{CAL}}]$, $\exists$ $\mathrm{n_{CALINT}}_i$ INT associated with CAL_i	$0 \leqslant \mathrm{n_{CALINT}}_i \leqslant \mathrm{n_{INT}}$	
	$\forall i \in [1, \mathrm{n_{CAL}}]$, $\exists$ $\mathrm{n_{CALDIF}}_i$ DIF associated with CAL_i	$0 \leqslant \mathrm{n_{CALDIF}}_i \leqslant \mathrm{n_{DIF}}$	

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Associat.	Axioms	Constraints	Comments
	$\forall i \in [1, \mathrm{n_{CAL}}]$, $\exists$ $\mathrm{n_{CALALG}}_i$ ALG associated with CAL_i	$0 \leqslant \mathrm{n_{CALALG}}_i \leqslant \mathrm{n_{ALG}}$	
10 $\longleftrightarrow$ 11	$\forall i \in [1, \mathrm{n_{PAR}}]$, $\exists$ $\mathrm{n_{PARALG}}_i$ ALG associated with PAR_i	$0 \leqslant \mathrm{n_{PARALG}}_i \leqslant \mathrm{n_{ALG}}$	
	$\forall i \in [1, \mathrm{n_{INT}}]$, $\exists$ 1 CI associated with INT_i	$\mathrm{n_{CI}} = \mathrm{n_M} + \mathrm{n_D} + \mathrm{n_{INT}}$	

The following remarks supplement the associations between the structure, the equations, and the variables displayed in Fig. 2.

Remark 1: BP_i relates the $\mathrm{n_{J1}}_i$ data P associated with $\mathrm{J1}_i$.

Remark 2: $\mathrm{LCB1}_i$ relates the $\mathrm{n_{J1}}_i$ data E associated with the $\mathrm{n_{J1}}_i$ data P of remark 1.

Remark 3: The $\mathrm{LCE1}_i$ relate the $\mathrm{n_{J1}}_i$ data F associated with the $\mathrm{n_{J1}}_i$ data P of remark 1.

Remark 4: $\mathrm{BP}_{\mathrm{n_{J1}}+i}$ relates the $\mathrm{n_{J0}}_i$ data P associated with $\mathrm{J0}_i$.

Remark 5: $\mathrm{LCB0}_i$ relates the $\mathrm{n_{J0}}_i$ data F associated with the $\mathrm{n_{J0}}_i$ data P of remark 4.

Remark 6: The $\mathrm{LCE0}_i$ relate the $\mathrm{n_{J0}}_i$ data E associated with $\mathrm{n_{J0}}_i$ data P of remark 4.

Remark 7: EP_i relates the 2 data P associated with TF_i.

Remark 8: LTE_{2i-1} relates the 2 data E associated with the 2 data P of remark 7.

Remark 9: LTE_{2i} relates the 2 data F associated with the 2 data P of remark 7.

Remark 10: $\mathrm{EP}_{\mathrm{n_{TF}}+i}$ relates the 2 data P associated with GY_i.

Remark 11: $\mathrm{LTE}_{2\mathrm{n_{TF}}+2i-1}$ relates the 2 data E and F associated with the 2 data P of remark 10.

Remark 12: $\mathrm{LTE}_{2\mathrm{n_{TF}}+2i}$ relates the 2 complementary data E and F (with respect to that of the previous remark) associated with the 2 data P of remark 10.

Remark 13: The data E and F are associated with the corresponding data P in the association 1 $\longleftrightarrow$ 2.

Remark 14: $\forall i \in [1, \mathrm{n_P}]$, $\mathrm{EN}_i = \int_0^t \mathrm{P}_i \mathrm{d}\tau + \mathrm{EN}_{i0}$. EN_{i0} corresponds to a piece of data CI which can be furnished if EN is pointed out by the user.

Remark 15: One of the data P associated with PPE_i is exclusively associated with it.

Remark 16: Several M and several D can be associated with the same piece of data EN.

Remark 17:

- if PPE_i is associated with a piece of data TYPE$\in${I,C,IC} and for the E and/or F associated with $\mathrm{n_{PPE}}_i$ of the $\mathrm{n_{PPE}}_i$+1 data P (*Cf.* association 2 $\longleftrightarrow$ 5):
  the $\mathrm{n_{PPE}}_i$ LC relate the $\mathrm{n_{PPEI}}_i$ data F to the $\mathrm{n_{PPEI}}_i$ data M associated with the corresponding E (*Cf.* association 8 $\longleftrightarrow$ 8), and/or the $\mathrm{n_{PPEC}}_i$ data E to the $\mathrm{n_{PPEC}}_i$ data D associated with the corresonding F (*Cf.* association 8 $\longleftrightarrow$ 8). $\mathrm{n_{PPEI}}_i + \mathrm{n_{PPEC}}_i = \mathrm{n_{PPE}}_i$, $\sum_{i=1}^{\mathrm{n_{PPE}}} \mathrm{n_{PPEI}}_i = \mathrm{n_M}$ and $\sum_{i=1}^{\mathrm{n_{PPE}}} \mathrm{n_{PPEC}}_i = \mathrm{n_D}$.
- if PPE_i is associated with the piece of data TYPE=R:
  the $\mathrm{n_{PPE}}_i$ LC relate the $\mathrm{n_{PPE}}_i$ data E and F associated with $\mathrm{n_{PPE}}_i$ of the $\mathrm{n_{PPE}}_i$+1 data P (*Cf.* association 2 $\longleftrightarrow$ 5).

These relations can possibly involve intermediary calculus (*Cf.* association 5 $\longleftrightarrow$ 11) as well as supplementary variables (*Cf.* association 5 $\longleftrightarrow$ 9).

Remark 18: LCBP relates the n_{PPEi}+1 data P associated with PPE_i (*Cf.* association 2⟷5).

Remark 19: If PPE_i is associated with a piece of data TYPE=I, C or IC, then $n_{PPEMODi} = 0$.

Remark 20: If PPE_i is associated with a piece of data TYPE=I,C or IC, then $n_{PPECIi} = n_{PPEINTi} = n_{PPEi}$ where n_{PPEi} is the port number of PPE_i.

Remark 21: $\sum_{i=1}^{n_{IN}} (n_{ININTi} + n_{INDIFi} + n_{INALGi}) = n_{IN}$.

Remark 22: $\sum_{i=1}^{n_{OUT}} (n_{OUTINTi} + n_{OUTDIFi} + n_{OUTALGi}) = n_{OUT}$.

5.2 Data associated with the structural analysis concepts

Structural analysis requires additional data. They are presented in table 5.

Characteristics of the structural analysis concepts: notion of order (*Cf. Sec. 2.3*):
It is important to introduce in the data model the notions of length and order since, on one hand, they may be used in the criteria for the choice of a set of I/O causal paths and, on the other hand, they will furnish the constraints on the output specifications. Consequently:

- each causal path (concept 13) is characterized by an integer corresponding to its length,
- each causal path is also characterized by an integer corresponding to its order,
- each set of causal paths (concept 15) is characterized by an integer corresponding to the sum of the lengths of the causal paths that compose the set,
- each output (concept 9) is characterized by an integer corresponding to its essential order.

Table 5: Table of the data associated with the structural analysis concepts

Concepts	Data	Constraints	Comments
Concept 12 Power lines	n_{LP} LP : $\{LP_1,\cdots,LP_{n_{LP}}\}$		*Cf. Sec. 2.2*
Concept 13 Causal paths	n_{CC} CC : $\{CC_1,\cdots,CC_{n_{CC}}\}$		*Cf. Sec. 2.3*
Concept 14 Sets of power lines	n_{ELP} ELP : $\{ELP_1,\cdots,ELP_{n_{ELP}}\}$		
15 Sets of causal paths	n_{ECC} ECC : $\{ECC_1,\cdots,ECC_{n_{ECC}}\}$		

Furthermore new associations are presented in Tab. 6.

6 Conclusion

This report specified the data model that supports the basic physical concepts and the physical structural analysis concepts used in the sizing methodology based on model inversion. Now the following step is to translate these specifications in programming ones for implementing:

Table 6: Axioms on the associations involving the structural analysis data

Associat.	Axioms	Constraints	Comments
2 ⟷ 12	$\forall i \in [1, n_{LP}]$, $\exists$ n_{LPi} P associated with LP_i	$n_{LPi} \leqslant n_P$	
8 ⟷ 13	$\forall i \in [1, n_{CC}]$, $\exists$ n_{CCEi} E associated with CC_i	$n_{CCEi} + n_{CCFi} \leqslant 2n_{VP}$	
	$\forall i \in [1, n_{CC}]$, $\exists$ n_{CCFi} F associated with CC_i		
	$\forall i \in [1, n_{CC}]$, $\exists$ n_{CCMi} M associated with CC_i	$n_{CCMi} \leqslant n_M$	
	$\forall i \in [1, n_{CC}]$, $\exists$ n_{CCDi} D associated with CC_i	$n_{CCDi} \leqslant n_D$	
9 ⟷ 13	$\forall i \in [1, n_{CC}]$, $\exists$ n_{CCMODi} MOD associated with CC_i	$n_{CCMODi} \leqslant n_{MOD}$	
	$\forall i \in [1, n_{CC}]$, $\exists$ n_{CCINi} IN associated with CC_i	$n_{CCINi} \leqslant 1$	
	$\forall i \in [1, n_{CC}]$, $\exists$ n_{CCOUTi} OUT associated with CC_i	$n_{CCOUTi} \leqslant 1$	
	$\forall i \in [1, n_{CC}]$, $\exists$ n_{CCCALi} CAL associated with CC_i	$n_{CCCALi} \leqslant n_{CAL}$	
12 ⟷ 14	$\forall i \in [1, n_{ELP}]$, $\exists$ n_{ELPi} LP associated with ELP_i	$n_{ELPi} \leqslant n_{LP}$	
13 ⟷ 15	$\forall i \in [1, n_{ECC}]$, $\exists$ n_{ECCi} CC associated with ECC_i	$n_{ECCi} \leqslant n_{CC}$	

- a formalism that is capable to carry on the previous concepts,
- a graphic editor that will help a designer manipulate these concepts in view of undertaking a physical structural analysis, applying the sizing methodology and thus, solving his corresponding design problem.

In the context of the methodology and in the Openprod project framework, bond graph and Modelica languages are privileged tools. There is a bond graph library whose the elements are described in Modelica: BondLib developed at ETH. This constitutes the starting point of the prototype developments in task T2.12.

As an illustration of the bond graph ability to fulfill the requirements presented in this document, Fig. 6 shows the acausal bond graph representation of the electrohydraulic example. It graphically displays all the basic physical concepts presented in Fig. 5. Furthermore it presents the power lines LP2 and LP3 identified in Sec. 2.2. Then Fig. 7 shows the causal bond graph representation where the set of the causal paths CC1 and CC2 are displayed. The lengths of these causal paths can easily be read with the number of I and/or C elements passed through. Finally the constraints of Sec. 4 are taken into account in the Fig. 8 bicausal bond graph whose the exploitation furnishes the corresponding inverse model.

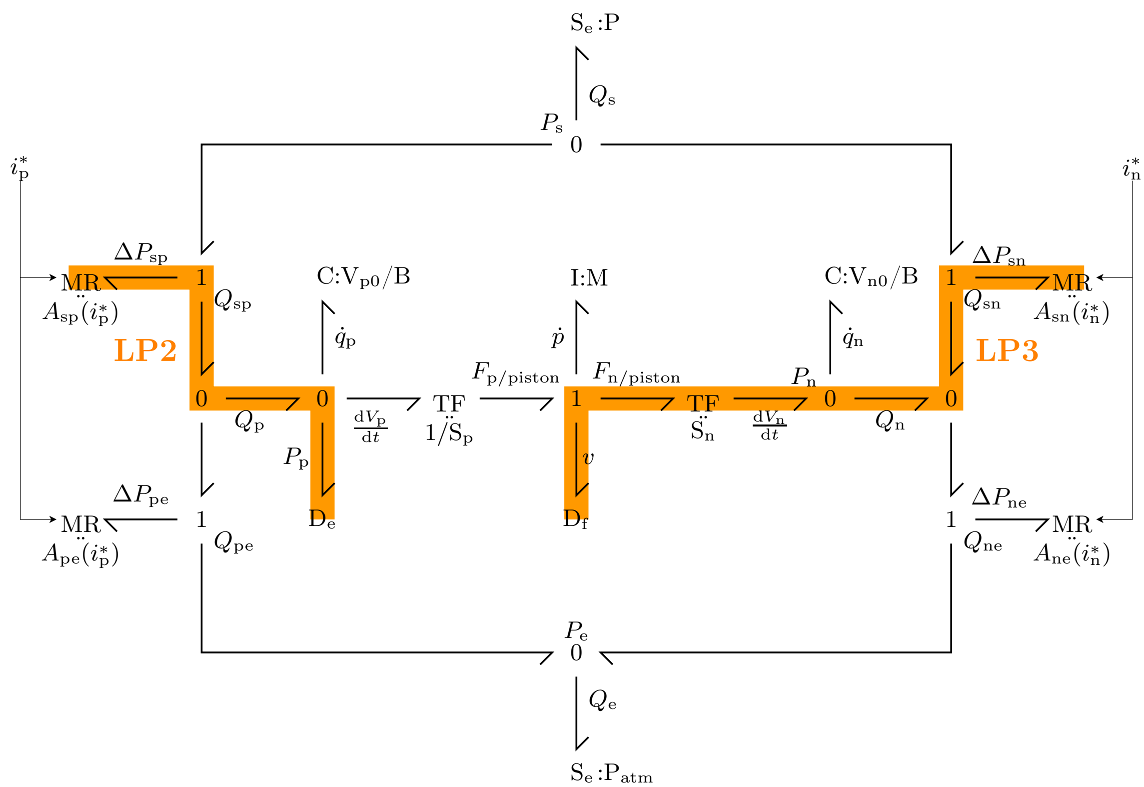

Figure 6: Acausal bond graph representation of the electrohydraulic example

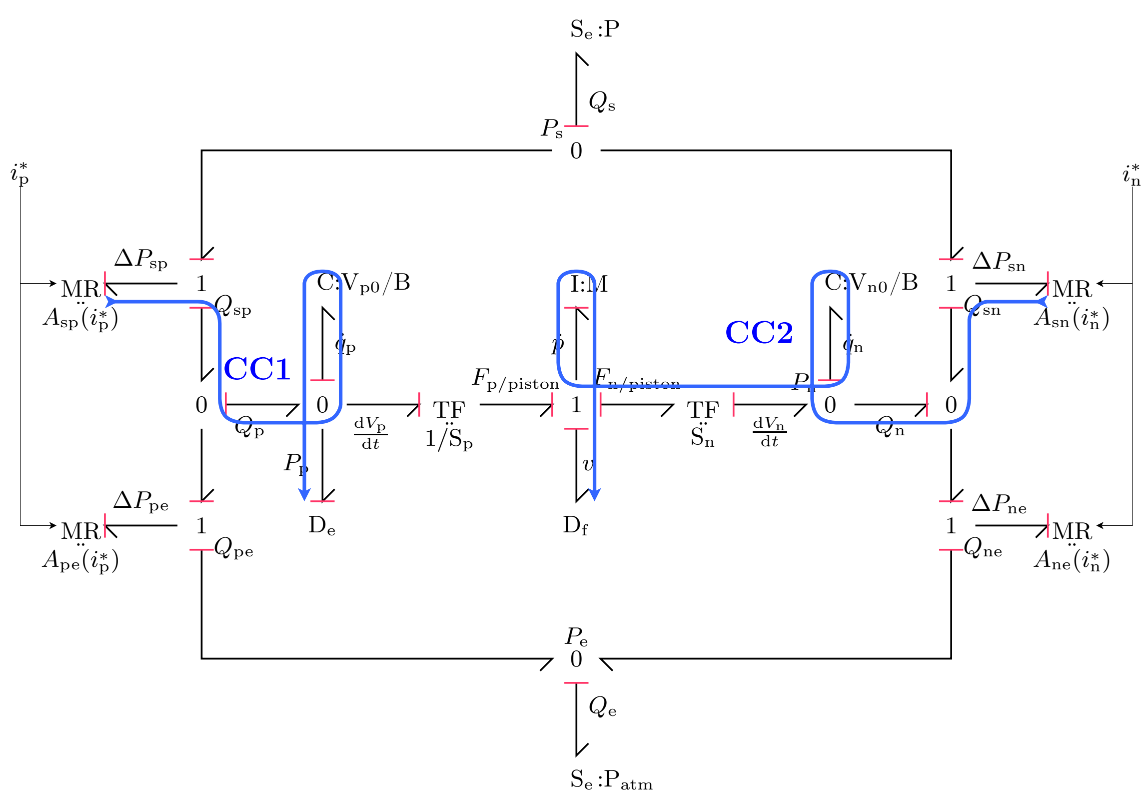

Figure 7: Causal bond graph representation of the electrohydraulic example

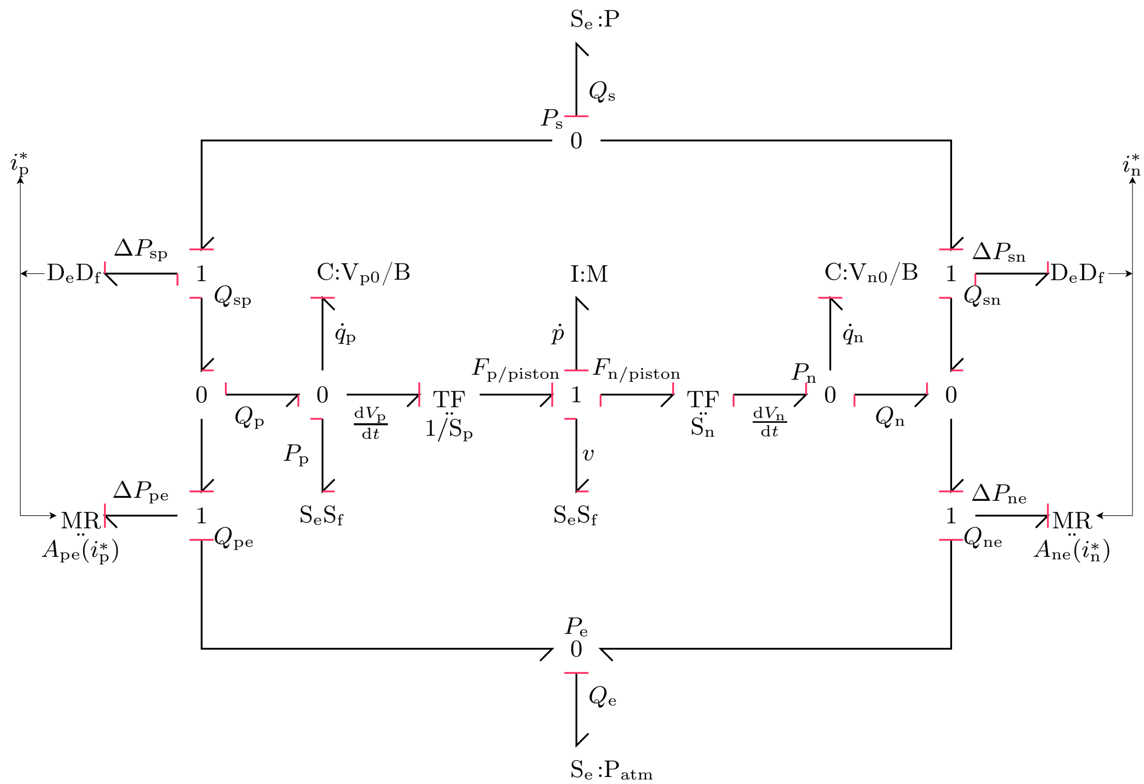

Figure 8: Bicausal bond graph representation of the electrohydraulic example

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Bibliography

[Bor92] Borne, P., Dauphin-Tanguy, G., Richard, J. P., Rotella, F., Zambettakis, I. *Modélisation et identification des processus : Tome 2*. Editions Technip, Paris, 1992. 309 p.

[Fot01a] Fotsu-Ngwompo, R., S. Scavarda, Thomasset, D. Physical model-based inversion in control systems design using bond graph representation. part 1: Theory. *Proceedings of the* IMECHE *Part I Journal of Systems and Control Engineering*, 215:95–103, 2001.

[Fot01b] Fotsu-Ngwompo, R., S. Scavarda, Thomasset, D. Physical model-based inversion in control systems design using bond graph representation. part 2: Applications. *Proceedings of the* IMECHE *Part I Journal of Systems and Control Engineering*, 215:105–112, 2001.

[Jar10] Jardin, A. *Contribution à une méthodologie de dimensionnement des systèmes mécatroniques : analyse structurelle et couplage à l'optimisation dynamique*. PhD thesis, Institut National des Sciences Appliquées de Lyon, Villeurbanne, France, 2010.

[Laf04] Laffite, J. *Aide à la conception et au dimensionnement énergétique et dynamique de systèmes mécatroniques par une approche inverse : Application aux chaînes de transmission automobiles*. Ph. d. thesis, Institut National des Sciences Appliquées de Lyon, 2004.